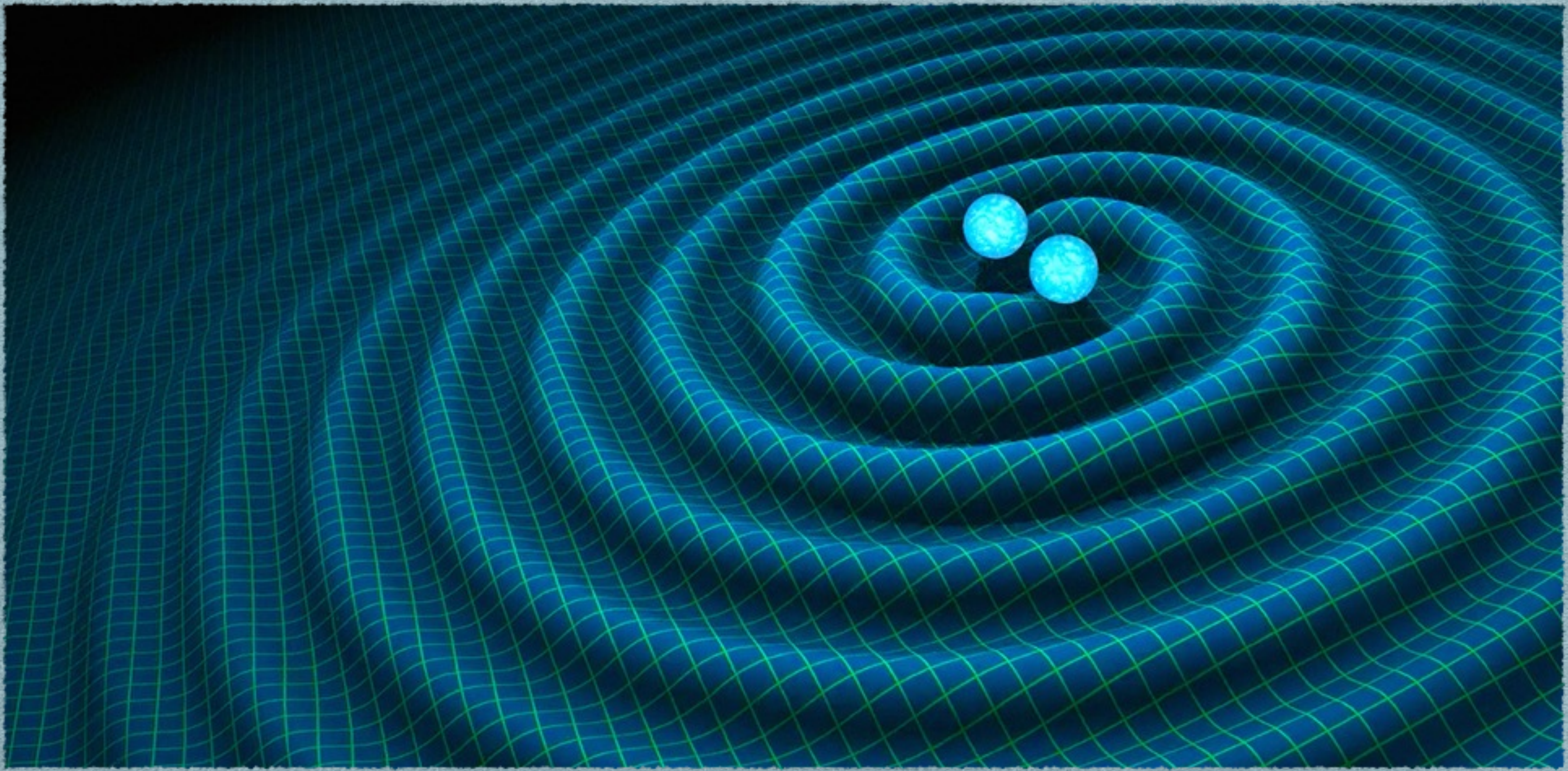


Einstein Made Personal:

How to Help Undergraduates “Own” General Relativity

Thomas A. Moore — June 6, 2016

Relativity's Reputation (1919)

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

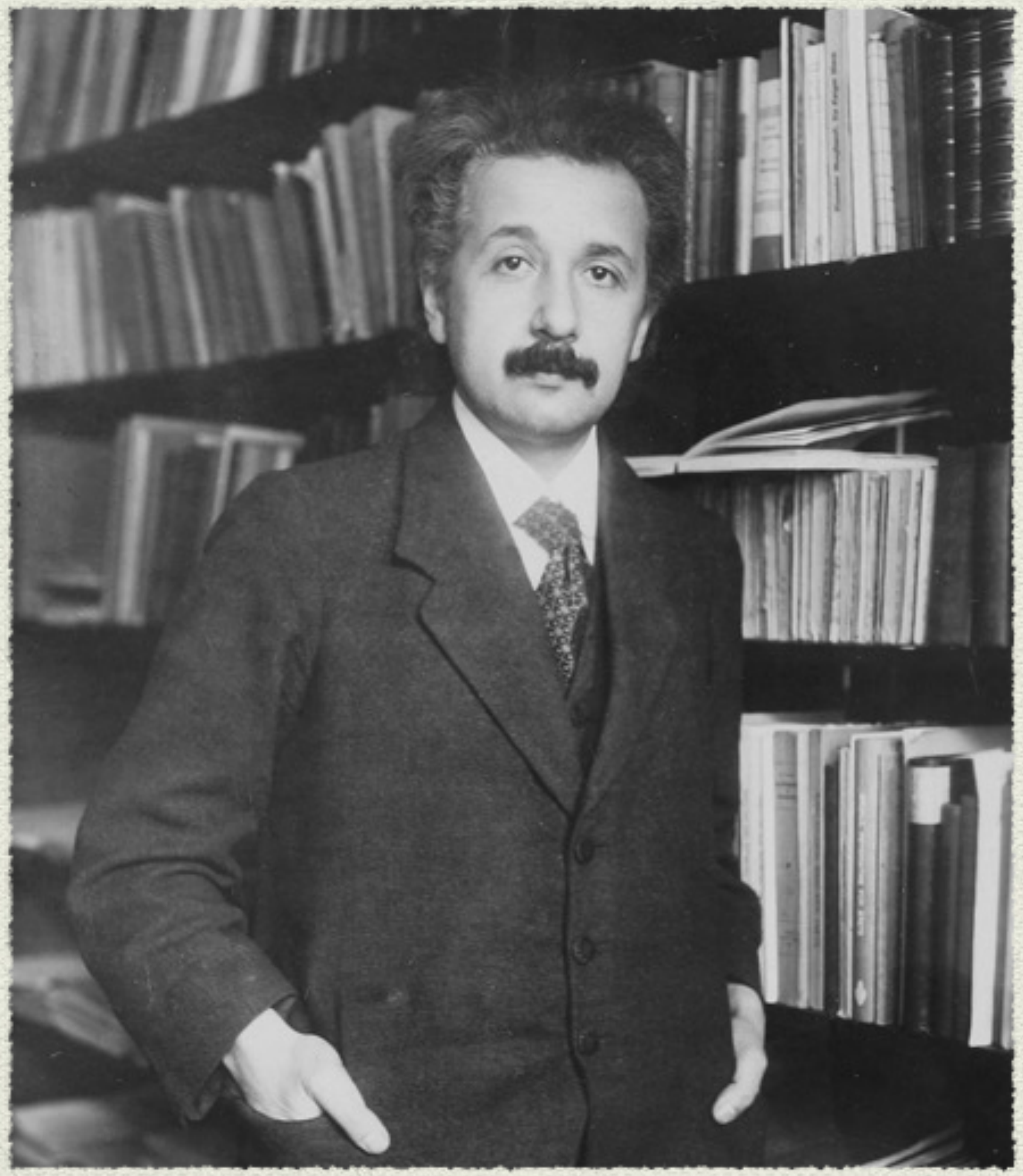
No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

Special Cable to THE NEW YORK TIMES.

LONDON, Nov. 9.—Efforts made to put in words intelligible to the non-

Einstein’s Reputation

- **Einstein: ‘Genius Among Geniuses’** (CNN, PBS)
- **Einstein’s Brain May Provide Clues To His Genius, Study Says** (Huffington Post)



Einstein in 1916

Ironically…

- Einstein's genius was his ability to make relativity (both SR and GR) *simple*

> There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did, because he was the only guy who caught on, before he wrote his paper. But after people read the paper a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that nobody understands quantum mechanics.

— Feynman, *The Character of Physical Law*

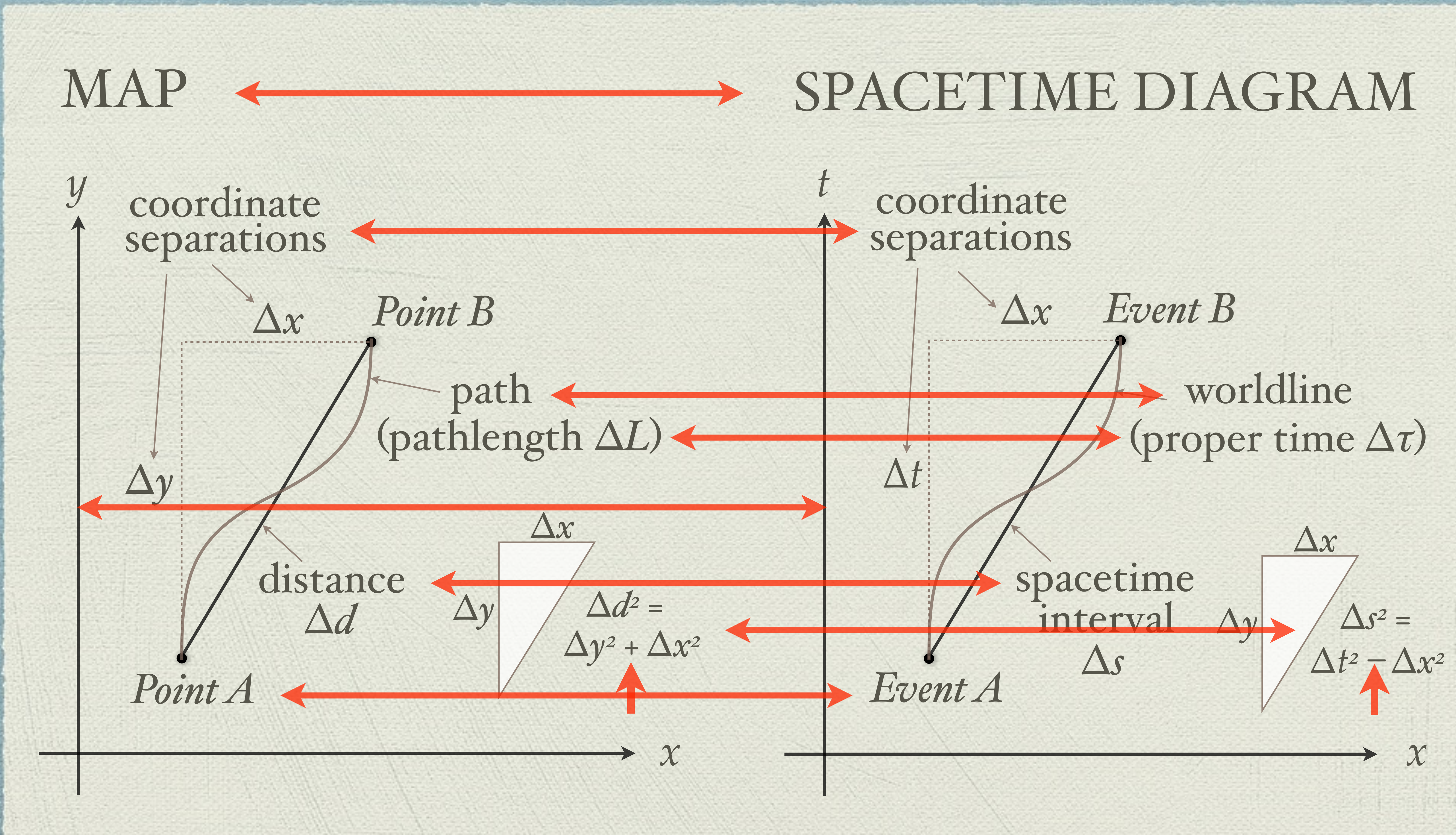
Problem and Opportunity

- The cultural context makes relativity both daunting and intriguing to students
- The good news is that undergraduates *can* successfully understand SR and GR (which is an empowering experience!)
- The keys are (1) sufficient *time,* (2) the right *tools,* and (3) *"teaching tasks"*

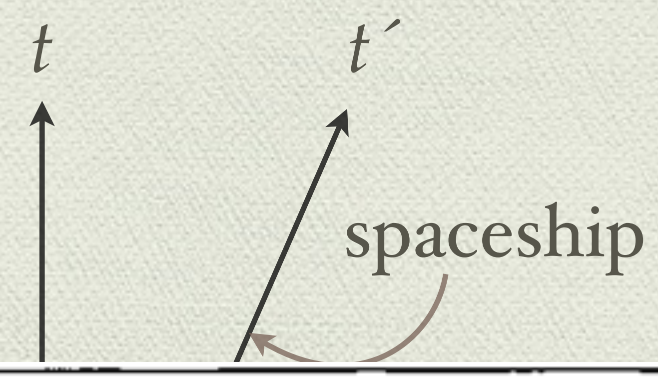
Special Relativity: *Time*

- Many introductory courses (if they discuss special relativity at all) spend three class sessions or less
- In our introductory course, we spend *nine* class sessions (3 full weeks)
- This provides sufficient time for students to process the new ideas

Special Relativity: *Tools*



Special Relativity: *Tools*



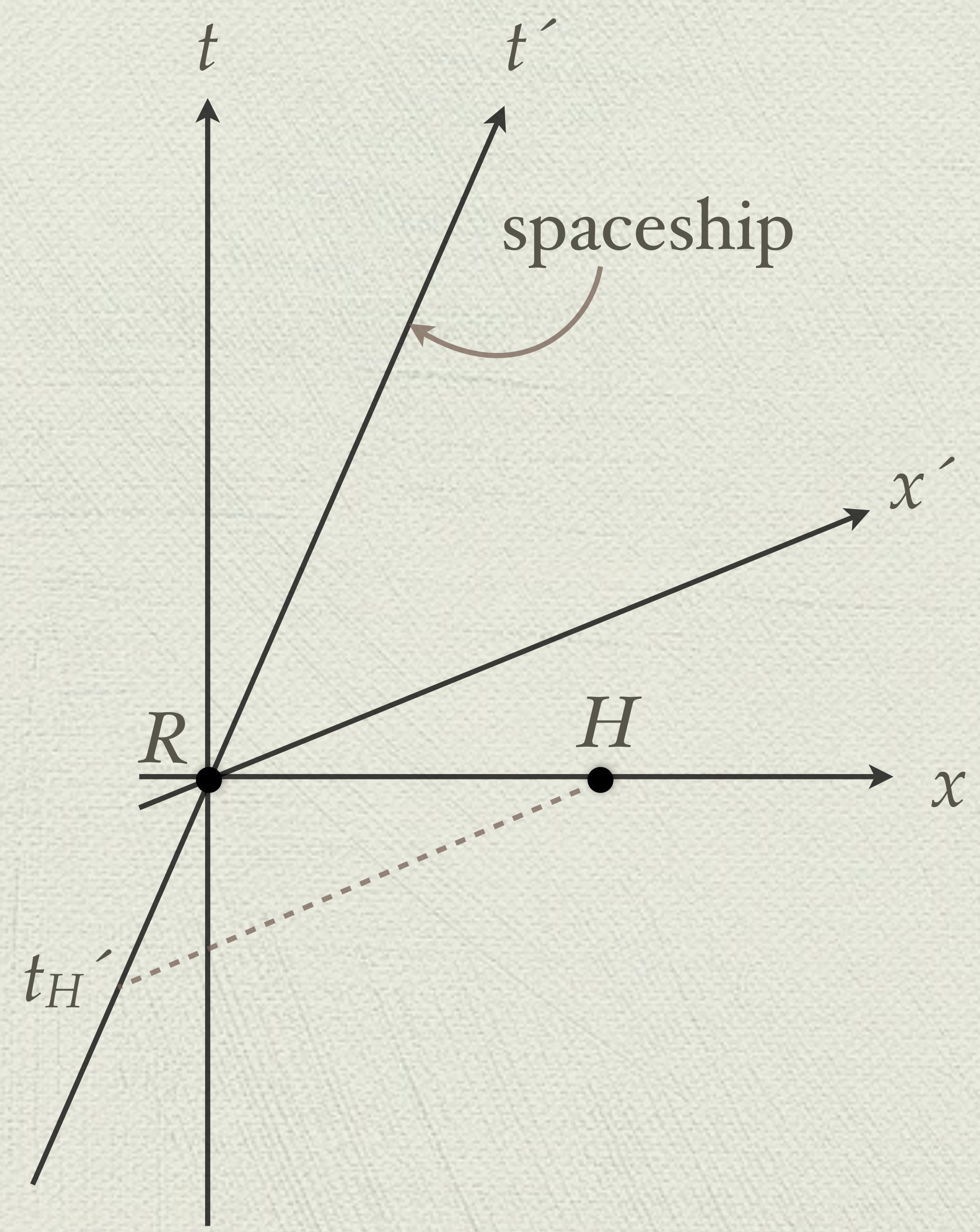
"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

Mt Rainier and Mt. Hood, which are 300 km apart in their rest frame, suddenly erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between the volcanoes. A fast spacecraft flying with constant speed $v = 0.8c$ from Rainier toward Hood is directly over Mt. Rainier when it erupts.

Let Event 1 be "Mt Rainier erupts," and Event 2 be "Mt. Hood erupts."

In the reference frame of the spacecraft, does Event 1 occur *before, after,* or *at the same time as* Event 2? Explain your reasoning.

Special Relativity: *Tools*



"Spacecraft problem" from Scherr, Schaffer, & Vokos, *AJP*, **70**, 12 (2002), pp. 1245-6.

Before tutorials: ~15%

After tutorials: 51%

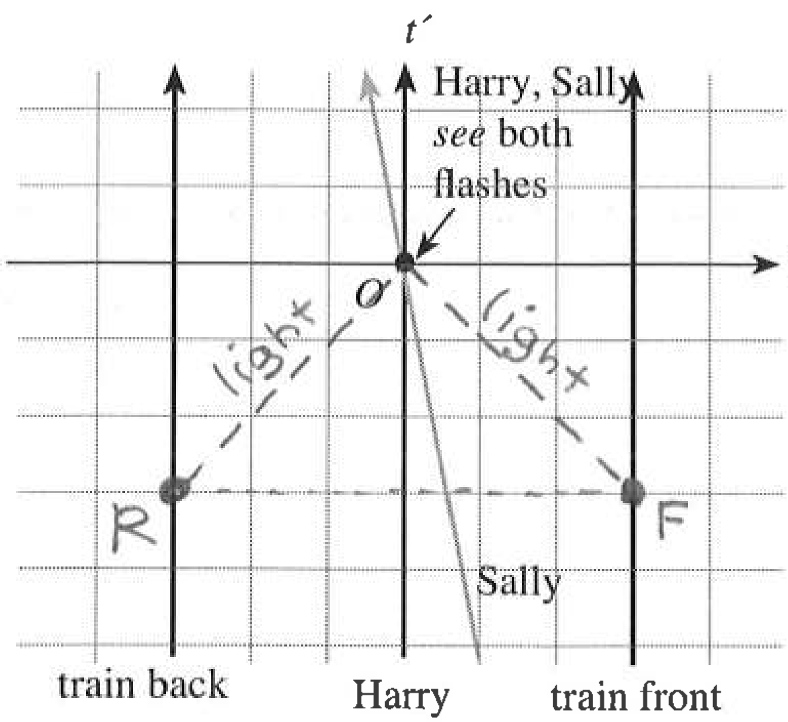
In my intro class:
~ 80%
(but only after hint to "draw a spacetime diagram"!)

Special Relativity: *Teaching Tasks*

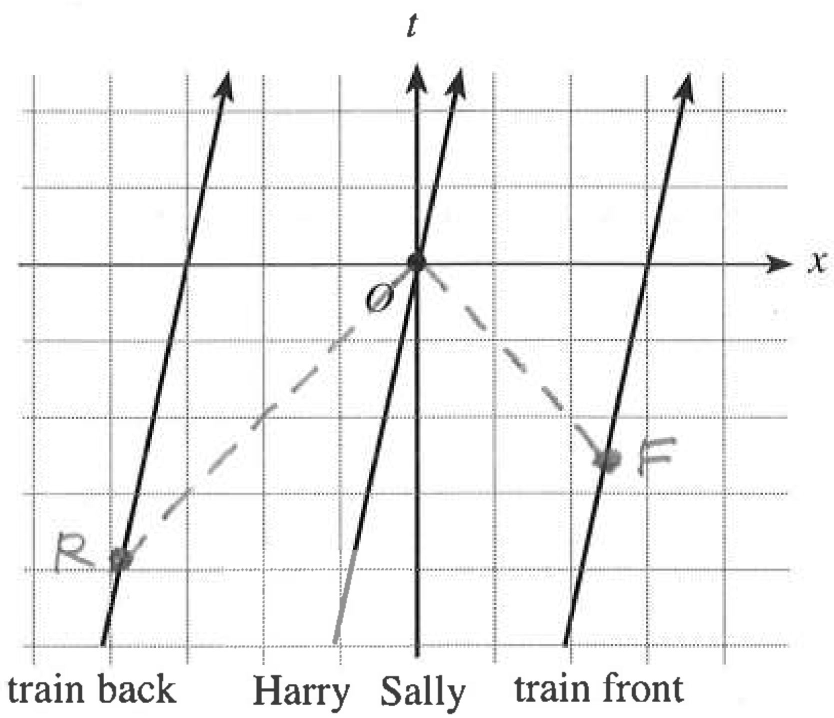
1. Practice Problem: **R3R.1**

(Example in-class worksheet)

View in Harry's Frame:



View in Sally's Frame:



Does Harry *observe* the lightning strikes to happen at the same time? Yes

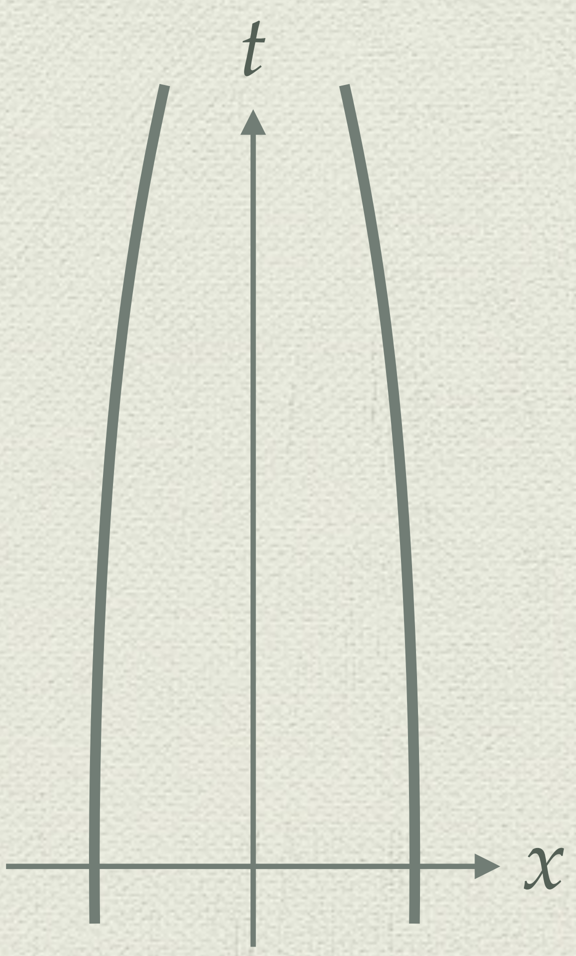
Does Sally *observe* the lightning strikes to happen at the same time? No (F is later)

Is the coordinate time separation between the strikes the same in both frames? No

(*Hint:* Draw in the light flash worldlines in both frames and see when & where they intersect the trains front and back ends.)

The Simplicity of GR

- **The Geodesic Hypothesis:** *Free objects follow geodesics in spacetime.* Consequences:
  - Free-float frames are inertial
  - The Equivalence Principle
  - Inertial mass = gravitational mass
- **Matter Curves Spacetime**
  - Observe tidal effects in a falling frame:
  - Initially parallel lines curve toward each other, so geometry is non-Euclidean!



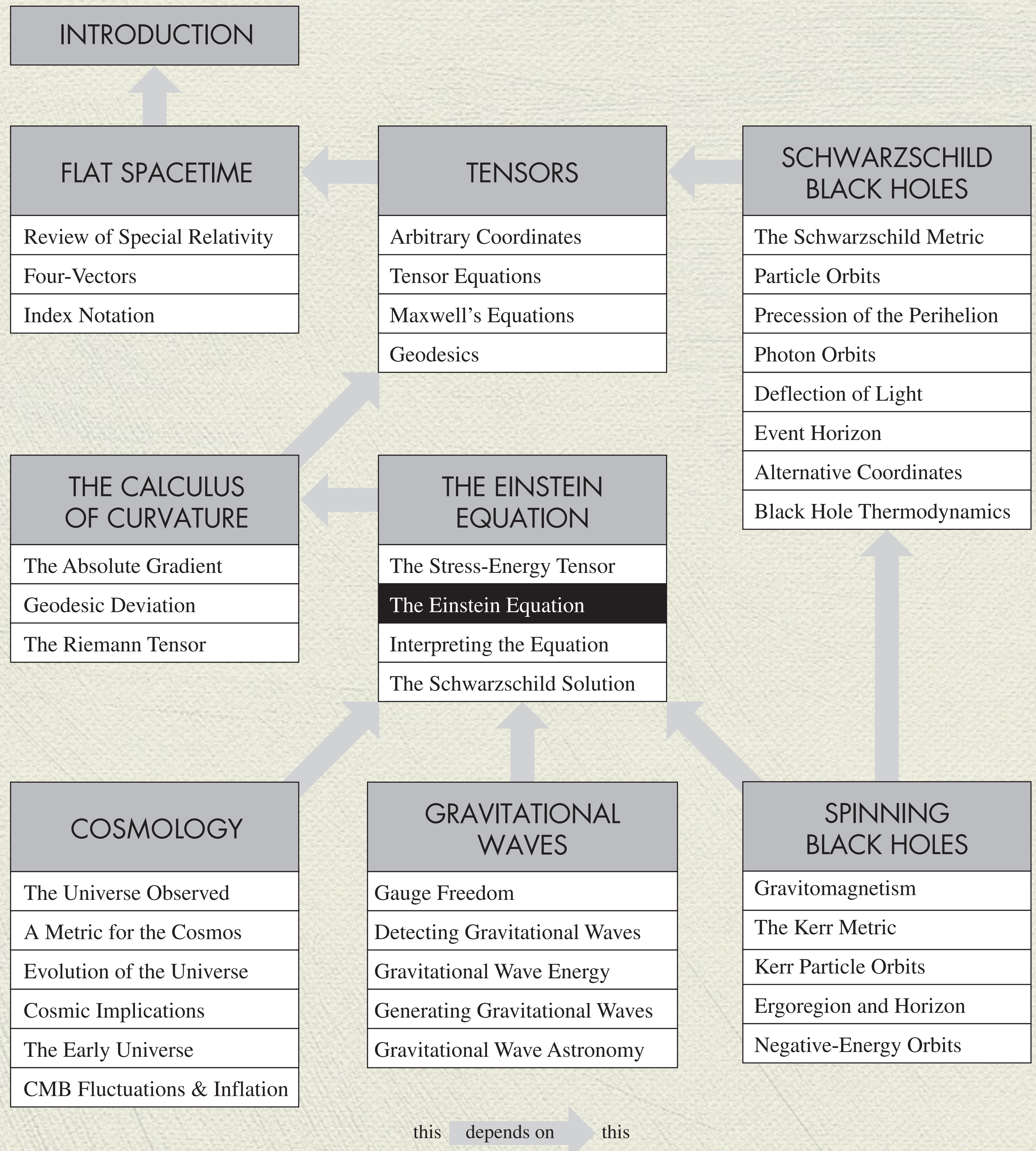
(See http://pages.pomona.edu/~tmoore/grw/Resources/GRWC1.pdf for details)

GR's Real Difficulty: The Math

- For undergraduates, tensor calculus is both *unfamiliar* and highly *abstract*
- Can we do without it?
  - Hartle, *Gravity*
  - Taylor and Wheeler: *Exploring Black Holes*
- My experience: students feel rootless
- Teaching requires *time, tools, teaching tasks*

General Relativity: *Time*

- My experience: a decent introduction to the Einstein equation and basic consequences for juniors and seniors requires *one full semester*
- One also should *spread out* the math
  - Two weeks on index notation and tensors in 2D and SR contexts
  - Three weeks exploring the geodesic equation
  - *Only then* talk about absolute derivatives, the Riemann tensor, and the Einstein equation


INTRODUCTION
FLAT SPACETIME
Review of Special Relativity
Four-Vectors
Index Notation
TENSORS
Arbitrary Coordinates
Tensor Equations
Maxwell's Equations
Geodesics
SCHWARZSCHILD BLACK HOLES
The Schwarzschild Metric
Particle Orbits
Precession of the Perihelion
Photon Orbits
Deflection of Light
Event Horizon
Alternative Coordinates
Black Hole Thermodynamics
THE CALCULUS OF CURVATURE
The Absolute Gradient
Geodesic Deviation
The Riemann Tensor
THE EINSTEIN EQUATION
The Stress-Energy Tensor
The Einstein Equation
Interpreting the Equation
The Schwarzschild Solution
COSMOLOGY
The Universe Observed
A Metric for the Cosmos
Evolution of the Universe
Cosmic Implications
The Early Universe
CMB Fluctuations & Inflation
GRAVITATIONAL WAVES
Gauge Freedom
Detecting Gravitational Waves
Gravitational Wave Energy
Generating Gravitational Waves
Gravitational Wave Astronomy
SPINNING BLACK HOLES
Gravitomagnetism
The Kerr Metric
Kerr Particle Orbits
Ergoregion and Horizon
Negative-Energy Orbits
this depends on this

General Relativity: *Tools*

- **Work with component notation**
  (because of connection to vectors)
- **Drill students on index notation**
- **Use lots of 2D examples in the beginning**
  - Polar coordinates
  - Funny flat-space coordinates
  - Longitude-latitude and other coordinates on the surface of a sphere

General Relativity: *Tools*

- **Dot-Product Method** for transforming to a Locally Orthonormal Frame (LOF)
  - $E = -\boldsymbol{u}_{\text{obs}} \cdot \boldsymbol{p} = -u^{\mu}_{\text{obs}} g_{\mu\nu} p^{\nu}$
  - $\boldsymbol{o}_t = \boldsymbol{u}_{obs}\, , \boldsymbol{o}_x\, , \boldsymbol{o}_y\, , \boldsymbol{o}_z\, ;\;\; \boldsymbol{o}_\alpha \cdot \boldsymbol{o}_\beta = \eta_{\alpha\beta}$
  - $a_{LOF,\alpha} = \boldsymbol{o}_\alpha \cdot \boldsymbol{a} = (\boldsymbol{o}_\alpha)^{\mu} g_{\mu\nu} a^{\nu}$
  - $T_{LOF,\,\alpha\beta} = (\boldsymbol{o}_\alpha)^{\mu} (\boldsymbol{o}_\beta)^{\nu} T_{\mu\nu}$
  - *Credit:* Hartle

General Relativity: *Tools*

- **Diagonal Metric Worksheet** for the metric

$$ds^2 = -A(dx^0)^2 + B(dx^1)^2 + C(dx^2)^2 + D(dx^3)^2$$

- **Off-Diagonal Metric Worksheet** for the metric

$$ds^2 = -A(dx^0)^2 + F dx^0 dx^1 + B(dx^1)^2 + C(dx^2)^2 + D(dx^3)^2$$

- See Moore, *AJP* **84**, 360 (2016) (original idea: Rindler)

$$\frac{1}{2B}\frac{d^2A}{dr^2}$$

$$R_{00} = 0 \quad + \frac{1}{2B}A_{11} \quad + \frac{1}{2C}\cancel{A_{22}} \quad + \frac{1}{2D}\cancel{A_{33}}$$

$$+\ 0 \quad - \frac{1}{2B}\cancel{B_{00}} \quad - \frac{1}{2C}\cancel{C_{00}} \quad - \frac{1}{2D}\cancel{D_{00}}$$

$$+\ 0 \quad + \frac{1}{4B^2}\cancel{B_0^2} \quad + \frac{1}{4C^2}\cancel{C_0^2} \quad + \frac{1}{4D^2}\cancel{D_0^2}$$

$$+\ 0 \quad + \frac{1}{4AB}\cancel{A_0}B_0 \quad + \frac{1}{4AC}\cancel{A_0}C_0 \quad + \frac{1}{4AD}\cancel{A_0}D_0$$

$$-\frac{1}{4BA}\left(\frac{dA}{dr}\right)^2 \quad - \frac{1}{4B^2}\frac{dA}{dr}\frac{dB}{dr} \quad + \frac{1}{4Br^2}2r\frac{dA}{dr} \quad + \frac{1}{4Br^2\cancel{\sin^2\theta}}2r\cancel{\sin^2\theta}\frac{dA}{dr}$$

$$-\frac{1}{4BA}A_1A_1 \quad - \frac{1}{4B^2}A_1B_1 \quad + \frac{1}{4BC}A_1C_1 \quad + \frac{1}{4BD}A_1D_1$$

$$-\frac{1}{4CA}\cancel{A_2}A_2 \quad + \frac{1}{4CB}\cancel{A_2}B_2 \quad - \frac{1}{4C^2}\cancel{A_2}C_2 \quad + \frac{1}{4CD}\cancel{A_2}D_2$$

$$-\frac{1}{4DA}\cancel{A_3}A_3 \quad + \frac{1}{4DB}\cancel{A_3}B_3 \quad + \frac{1}{4DC}\cancel{A_3}C_3 \quad - \frac{1}{4D^2}\cancel{A_3}D_3$$

$$\boxed{R_{00} = \frac{1}{2B}\left[\frac{d^2A}{dr^2} - \frac{1}{2A}\left(\frac{dA}{dr}\right)^2 - \frac{1}{2B}\frac{dA}{dr}\frac{dB}{dr} + \frac{2}{r}\frac{dA}{dr}\right]}$$

General Relativity: *Teaching Tasks*

- **Class Structure**
  - Book has overview/exercise structure
  - Students do all the exercises *before* class
  - During class, students present their work (using a document projector) for the hardest
  - If time remains (about half the time), we work on some example problems
  - I collect 2 books at the end of class to review
  - Weekly homework on applications

(An example of exercise references in the text)

If we plug $R_{tt} = R_{rr} = 0$ into equation 23.7, we find that

$$0 = \frac{B}{A}R_{tt} + R_{rr} = \frac{1}{r}\left(\frac{1}{A}\frac{\partial A}{\partial r} + \frac{1}{B}\frac{\partial B}{\partial r}\right) \quad \Rightarrow \quad \frac{1}{A}\frac{\partial A}{\partial r} = -\frac{1}{B}\frac{\partial B}{\partial r} \tag{23.9}$$

You can then plug this result into equation 23.6*c* to eliminate the references to A, solve the resulting equation for $\partial B/\partial r$, and integrate. Since B is independent of t, the result (see box 23.3) is

$$\frac{1}{B} = 1 + \frac{C}{r} \tag{23.10}$$

where C is a constant of integration.

Since $\partial B/\partial t = 0$ by equation 23.8, the right side of 23.9 is independent of time. The left side must therefore be as well. This will be true only if any time dependence in A has the form $A(t,r) = f(t)a(r)$, because in that case,

$$-\frac{1}{B}\frac{dB}{dr} = \frac{1}{A}\frac{\partial A}{\partial r} = \frac{1}{\cancel{f(t)}a(r)}\cancel{f(t)}\frac{da}{dr} = \frac{1}{a}\frac{da}{dr} \tag{23.11}$$

(note that I am using dB/dr and da/dr instead of $\partial B/\partial r$ and $\partial a/\partial r$ because I know that B and a depend only on r). Since we know what $B(r)$ is, we can solve equation 23.11 for $a(r)$: the result (see box 23.4) is

$$a = \frac{K}{B} = K\left(1 + \frac{C}{r}\right) \tag{23.12}$$

(An example of an exercise box in the text)

BOX 23.3 Solving for B

Equation 23.9 (repeated here) tells us that

$$\frac{1}{A}\frac{\partial A}{\partial r} = -\frac{1}{B}\frac{\partial B}{\partial r} \tag{23.9r}$$

According to equation 23.6*c*, the condition that $R_{\theta\theta} = 0$ implies that

$$0 = -\frac{r}{2AB}\frac{\partial A}{\partial r} + \frac{r}{2B^2}\frac{\partial B}{\partial r} + 1 - \frac{1}{B} \tag{23.23}$$

If you use equation 23.9 to eliminate the factors of A from equation 23.23, you should find that the latter reduces to

$$1 = -\frac{r}{B^2}\frac{\partial B}{\partial r} + \frac{1}{B} = \frac{\partial}{\partial r}\left(\frac{r}{B}\right) \tag{23.24}$$

Exercise 23.3.1. Show that equation 23.24 is correct.

Does it work?

- My students cover more and perform better on HW and exams than before
- Many distant users have told me how valuable they have found the book
- **Problems:** Students can find the work relentless. I also need a better way to deal with algebraically intensive boxes.

Does it work?

C. SUMMARY

Please respond generally to the following questions. If you need more space, you may use the other side of this page.

1. What has this faculty member done especially well?

Moore has done a wonderful job taking this difficult topic normally taught in grad school and making it accessable to undergrads. The structure of the course was fantastic for working through the material. I really enjoyed how the boxes allowed/encouraged/forced us to engage w/ the material before class. And then going over them in class provided extra reinforcement.

Summary

- With the appropriate *time*, *tools*, and *teaching tasks*, undergraduates at all levels can genuinely understand Einstein's theories

- They thereby come to better understand Einstein's *true* genius in seeing what is simple behind what seems superficially complex.

- This helps them more deeply appreciate current discoveries and celebrate that after 100 years,

Einstein
is still
da man!

Thank you!

More about *Six Ideas That Shaped Physics:*
www.physics.pomona.edu/sixideas/

More about *A General Relativity Workbook:*
pages.pomona.edu/~tmoore/grw/

In particular, the 1st chapter (which provides an overview of GR suitable for physics majors) is at http://pages.pomona.edu/~tmoore/grw/Resources/GRWC1.pdf