Numerical Relativity Simulations of Black Hole Mergers

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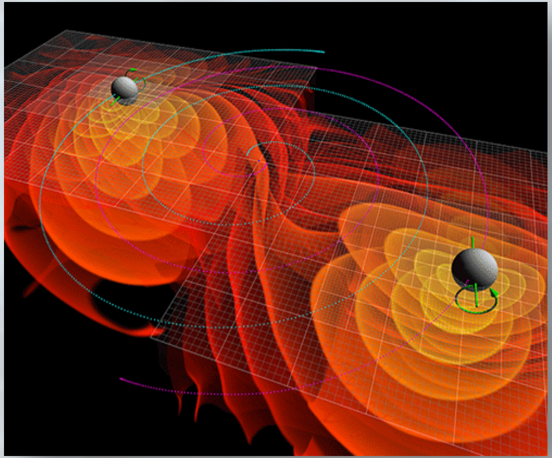
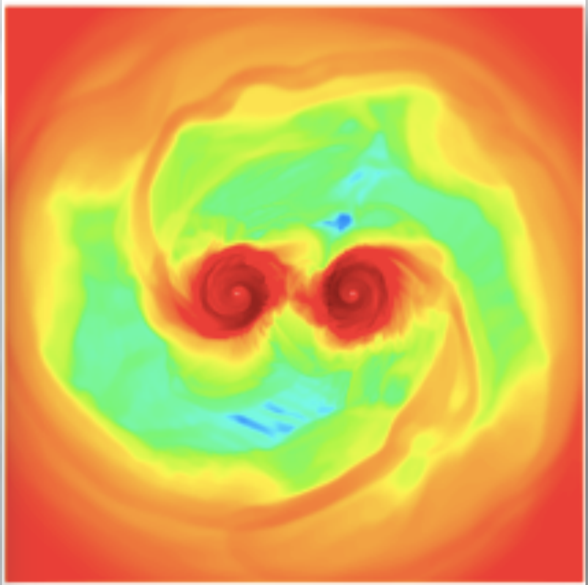
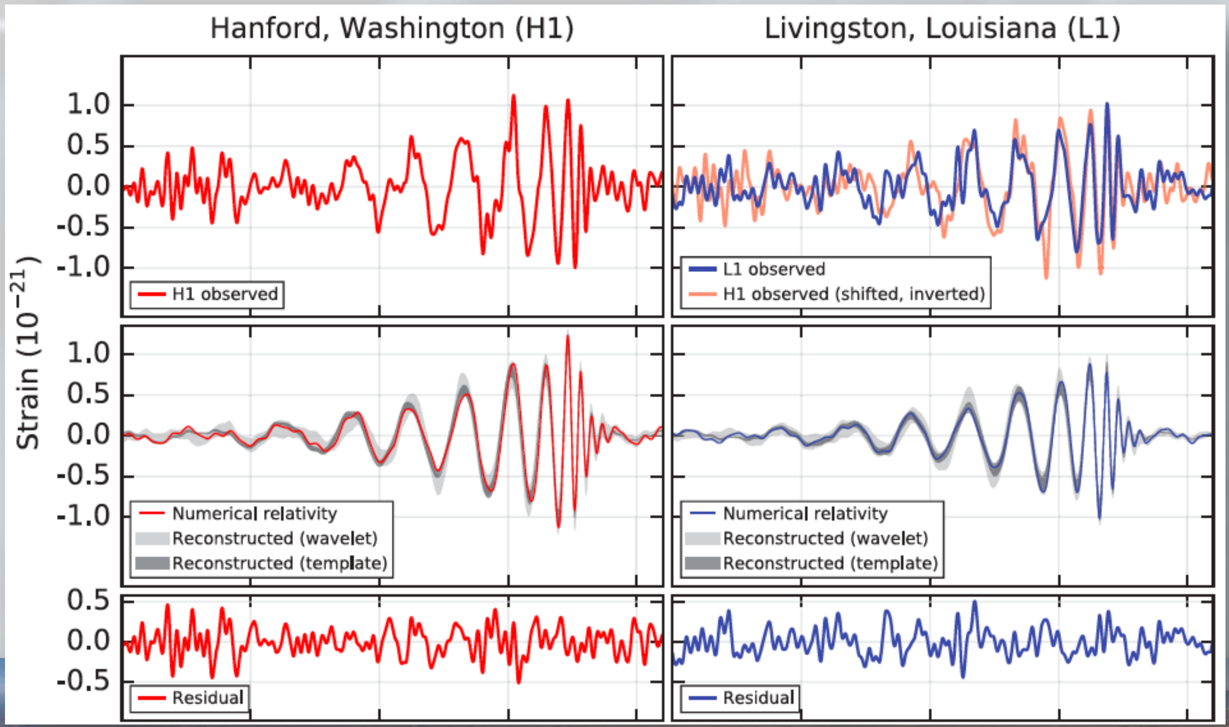
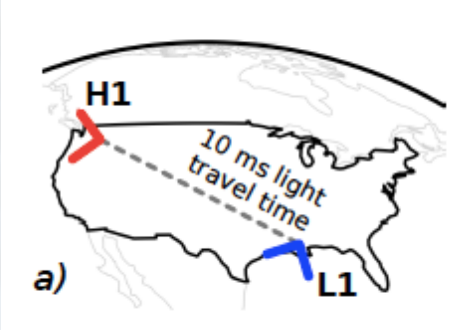




There are Binary Black Hole Mergers in the Universe

Gravitational waves from a BBH merger observed by the two aLIGO detectors on September 14, 2015





GW150914:

36 + 29 $M_{\odot}$

1.3 millions light years away

(z = 0.09)

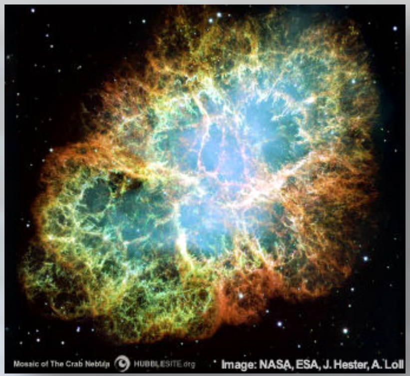
Ligo Scientific Collaboration, Phys. Rev. Lett. **116**, 061102 (2016)

LSC

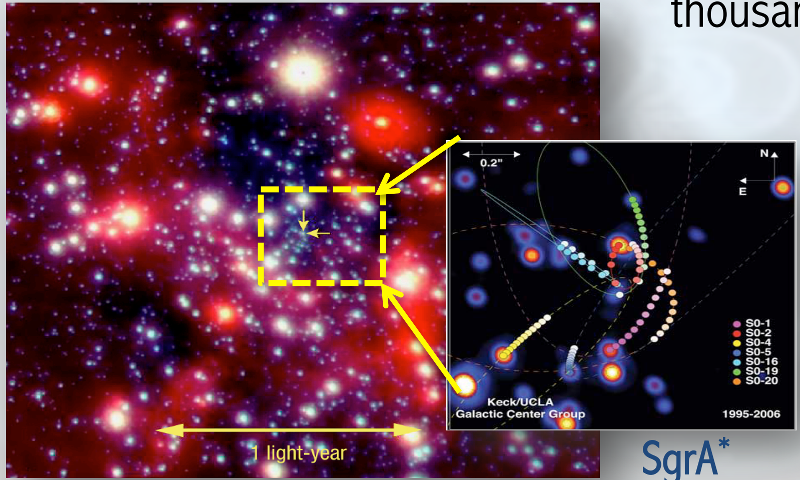
Black Holes Astrophysics Basics

When the largest stars, more than 20 times the mass of the sun ($M_\odot$), run out of fuel they collapse under their own gravity and form **stellar-mass black holes** …

X-ray binaries, e. g. Cygnus X-1



Due to dense stellar cluster dynamics these black holes (BHs) can grow by swallowing stars, gas, and other BHs and form **intermediate-mass black holes** which can have thousands of solar masses.

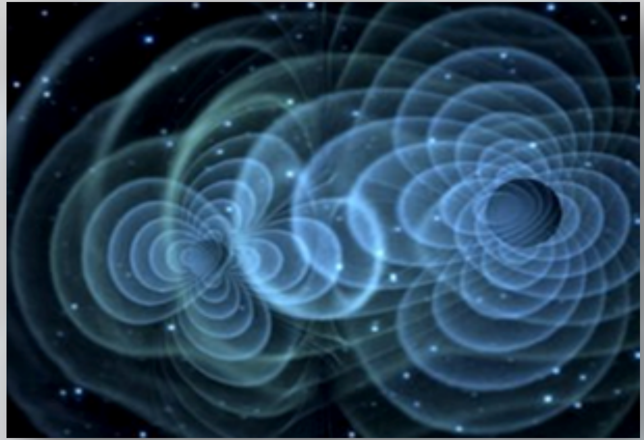


SgrA*

Supermassive black holes (SMBHs), a million to 10 billion times the mass of the Sun, lurk in the nuclei of almost all galaxies, including our own.

The evolution and dynamics of galaxies are strongly connected to these black holes.

Binary Black Holes Coalescences

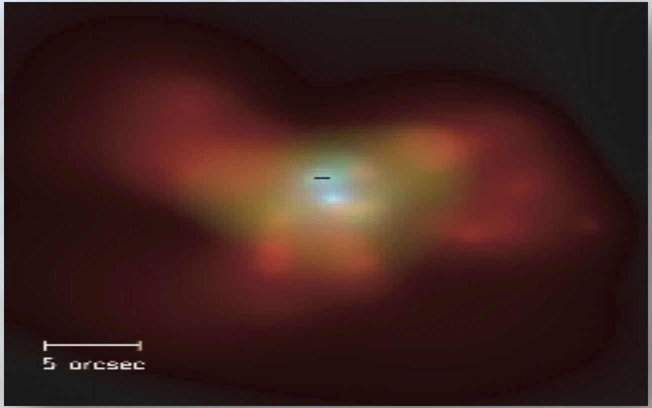


3D visualization of gravitational waves produced by 2 orbiting black holes. [Image: Henze, NASA]

When binary black holes (BBH) collide they spiral inward and merge in a violent explosion of gravitational waves.

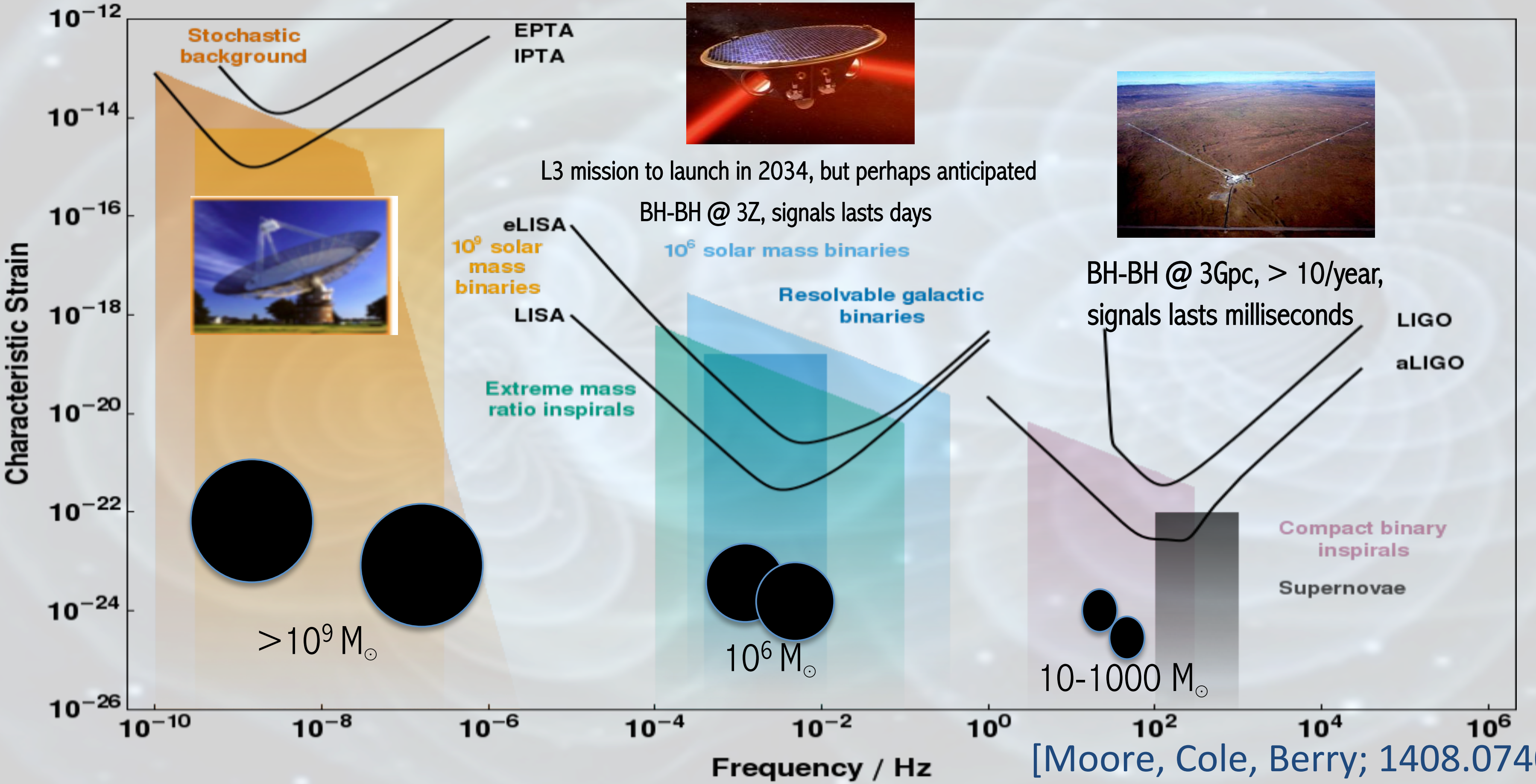
For supermassive black holes, the merger releases as much as 10^{61} ergs, the energy 10,000,000,000 suns release over their entire lifetimes.

This happens when galaxies, and consequently their central black hole collides collide.
Within galaxies themselves smaller stellar-size black holes are also thought to merge, producing intense gravitational waves.



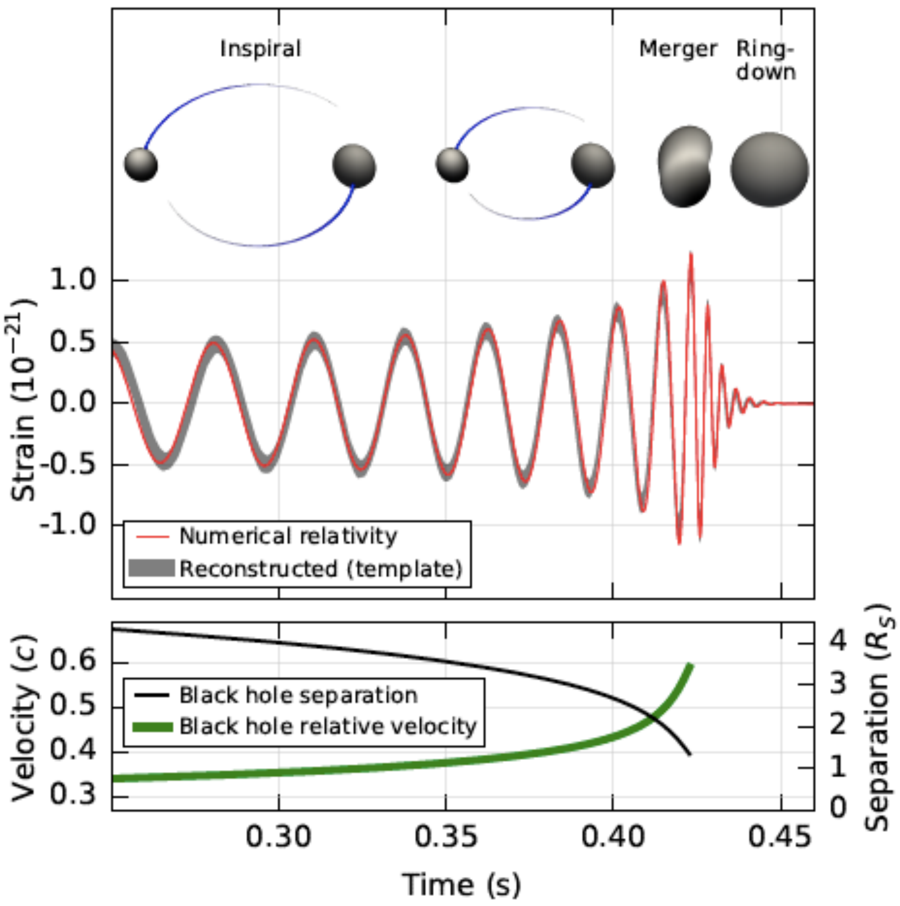
Dual black holes at the core of a galaxy, observed by Chandra, Komossa et al. 2003

Ideal Sources for Gravitational Wave Detectors



[Moore, Cole, Berry; 1408.0740]

Binary Black Holes in General Relativity



Inspiral ⟶ Post-Newtonian (PN) expansion when $v << c$

Merger ⟶ Numerical relativity (NR) when $v \sim c$

Ringdown ⟶ BH perturbation theory or NR

NR is needed to accurately to study the "late" Inspiral and Merger dynamics of BBHs, compute the gravitational waveforms, but also to determine what "late" means …

Gravitational waveforms encode information about the source that generated them: masses, spins, orbital parameters, etc.

Solving the Einstein's Equations

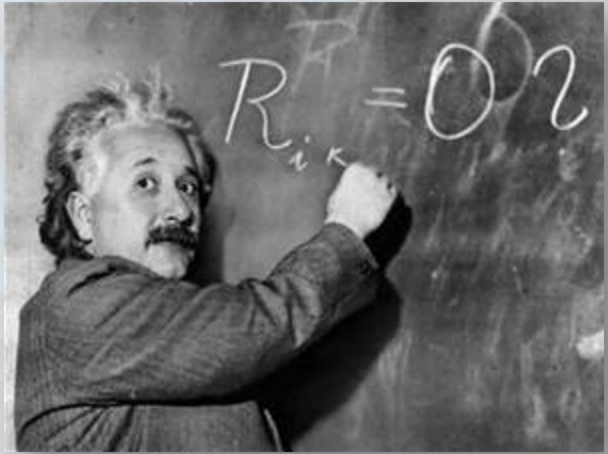
Numerical Relativity (NR) is about solving the Einstein's Field Equations numerically without any approximation.

Space-Time Curvature = Matter-Energy

$$G_{\mu\nu} = 8\pi\, T_{\mu\nu}$$

These equations form a set of 10 nonlinear, coupled, partial differential equations in 4 dimensions, that in general cannot be solved analytically, except:

- Static, spherical symmetric BH, Schwarzshild (1916)
- Stationary, axially symmetric, spinning BH Kerr (1961)
- Cosmology: isotropic and/or homogenous (1920s)



Written in a general coordinate system they have thousands of terms!

Geometrized Units c = G = 1: $r_G = G/c^2 M \sim M$, and $M \sim 5 \times 10^{-6}\ (M/M_{sun})$sec $\sim 1.5\ (M/M_{Sun})$ km

Conceiving Numerical Relativity

To find out what happens when black hole collide, we must solve the Einstein's Field Equations numerically …

MISNER summarized the discussion of this session: "First we assume that you have a computing machine better than anything we have now, and many programmers and a lot of money, and you want to look at a nice pretty solution of the Einstein equations. The computer wants to know from you what are the values of $g_{\mu\nu}$ and $\frac{\partial g_{\mu\nu}}{\partial t}$ at some initial surface, say at $t = 0$. Now, if you don't watch out when you specify these initial conditions, then either the programmer will shoot himself or the machine will blow up. In order to avoid this calamity you must make sure that the initial conditions which you prescribe are in accord with certain differential equations in their dependence on x, y, z at the initial time. These are what are called the "constraints." They are the equations analogous to but much more com-

GR 1: Conference on the role of gravitation in physics
University of North Carolina, Chapel Hill [January 18-23, 1957]

This is not an easy task!

It required 50+ years of efforts …

Collision of 2 “wormholes” and Geometrodynamics [Hahn & Lindquist]
ADM Formulation [Arnowitt, Deser & Misner]

1960s

1970s

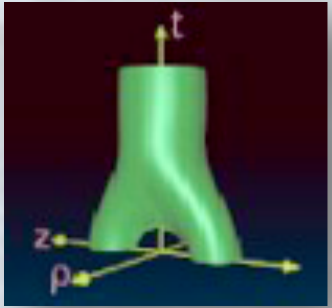
Pioneering efforts on supercomputers at Livermore Natl Lab [Smarr & Eppley]
Creation of NCSA [Smarr, NSF]

1980s

Unruh’ BH Excision [Unruh], New Formulation [York]

1990s

Large NSF projects:
1) LIGO moves ahead
2) Grand Challenge creates the first 3D code …



AEI group [Seidel]
Modern Formulations BSSN [Baumgarte & Shapiro]
New gauges [Alcubierre]
Lazarus Project [Baker, Campanelli, Lousto]
One orbit barrier [Bruegman et al]

2000s

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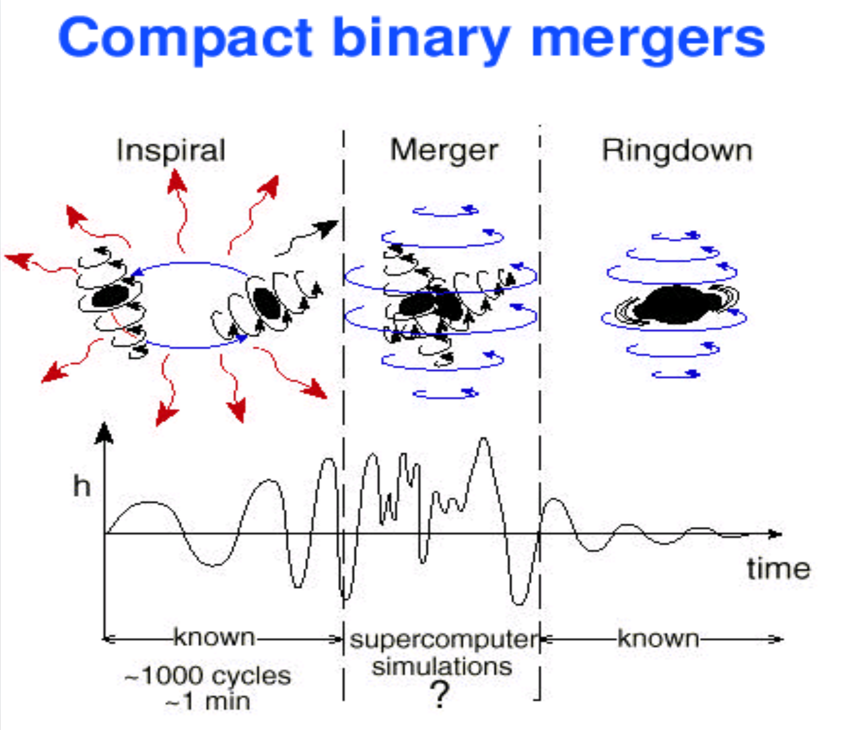
Kip Thorne's Bet

"I have bet these numerical relativists that gravitational waves will be detected from black-hole collisions before their computations are sophisticated enough to simulate them. I expect to win…"

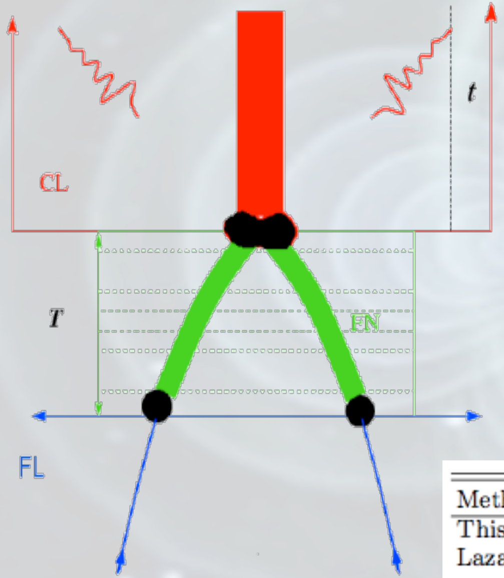


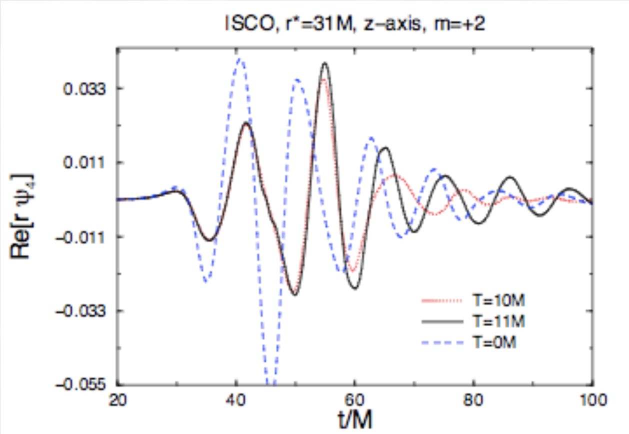
Reference: K.S. Thorne, [Spacetime Warps and the Quantum World: Speculations About the Future," in R.H. Price, ed., *The Future of Spacetime* (W.W. Norton, New York, 2002).]

"…. but hope to lose, because the simulation results are crucial to interpreting the observed waves."

The Lazarus Approach

Motivated by the close limit approximation to binary black hole merger [Price & Pullin, 1994], we used crashing NR simulations jointly with PN and perturbation theory to extend the lifetime of NR codes, Baker et al, Phys. Rev. Lett. 2001

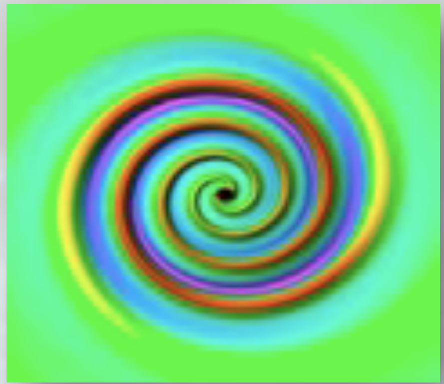




Method	E_{rad}/M	J_{rad}/M^2	Merger time/M	$a/M_{\mathcal{H}}$
This paper	2.8 ± 0.2	$14 \pm 1\%$	$T_{CAH} \approx 19$	0.683 ± 0.006
Lazarus[a]	2.5 ± 0.2	$13 \pm 2\%$	$T_{Tran} \approx 10$	0.70 ± 0.02

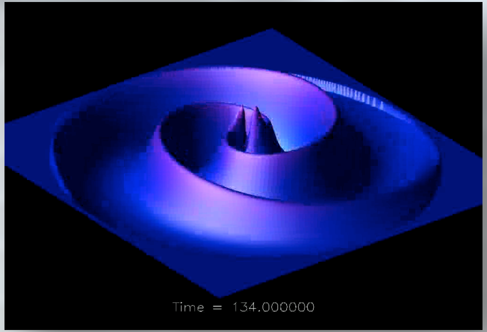
Binary Black Hole Problem "Solved"

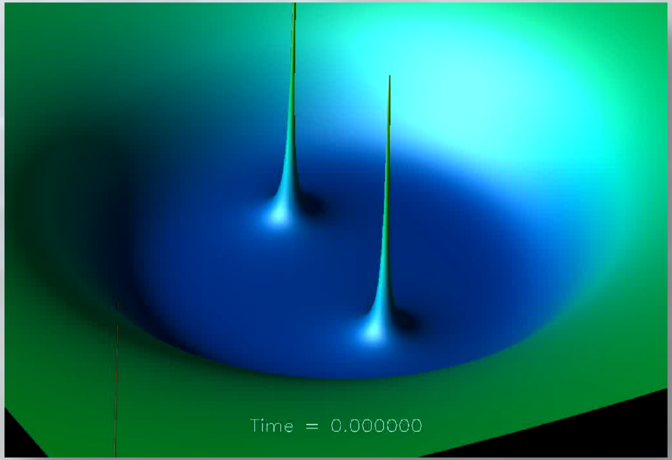
Binary inspiral and merger,
Pretorius, Phys.Rev.Lett. 95 (2005)



Black holes free to move across the grid!
Use excision or singularity avoiding slicing.
Coordinates should not be too distorted.
Gridpoints should resolve the region of interest.

Moving Punctures
Campanelli, Lousto, Zlochower, Marronetti, Phys.Rev.Lett. 96 111101 (2006)





Baker, Centrella, Choi, Koppitz, van Meter, Phys.Rev.Lett. 96 (2006)

'E pur si muove' - Galileo Galilei

It required 50+ years of efforts …

1960s

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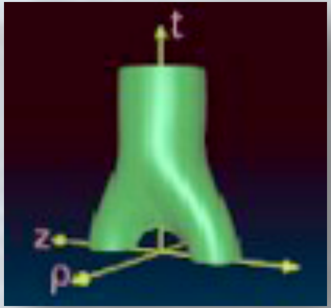

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New gauges [Alcubierre]

Lazarus Project [Baker, Campanelli, Lousto]

One orbit barrier [Bruegman et al]

2005

Breakthrough [Pretorius, Baker+, Campanelli+]

2010s

Moving Punctures Groups [EinsteinToolkit]

SPEC code [SXS Collaboration]

MOUs with LSC [Ninja and NRAR Collaborations]

Building Catalogs of Waveforms

Large collection of more than one thousand of NR simulations of binary black holes are available.

Center for Computational Relativity and Gravitation R·I·T

Simulating eXtreme Spacetimes

Cardiff University Prifysgol Caerdydd

Georgia Tech Center for Relativistic Astrophysics College of Sciences

UIB Universitat de les Illes Balears

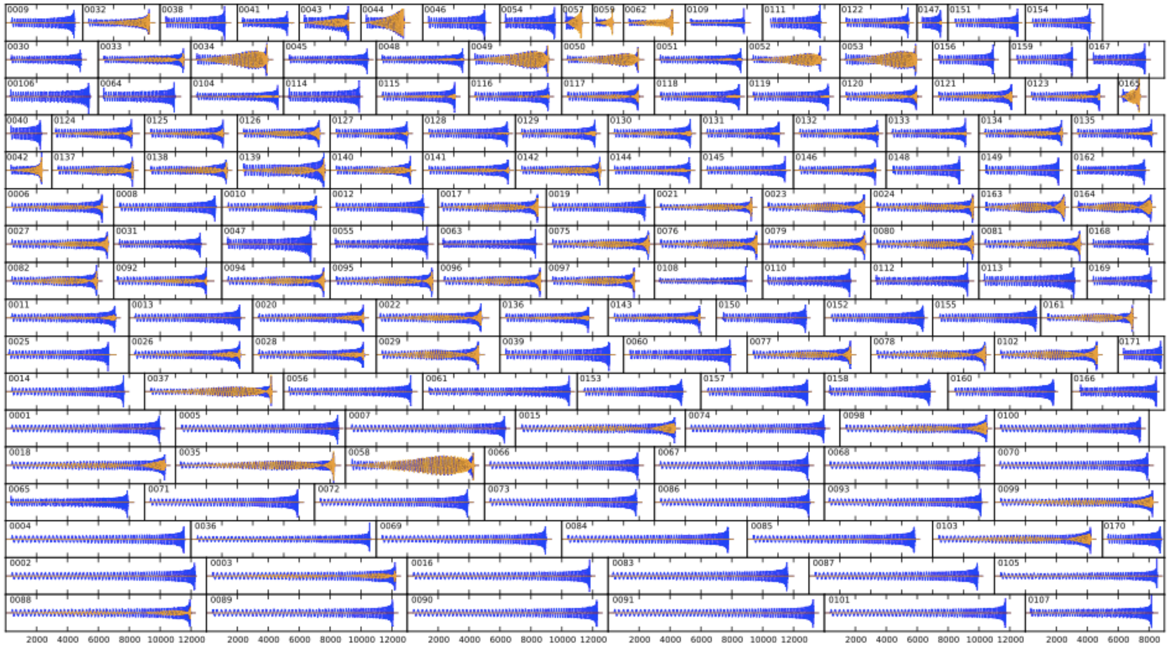
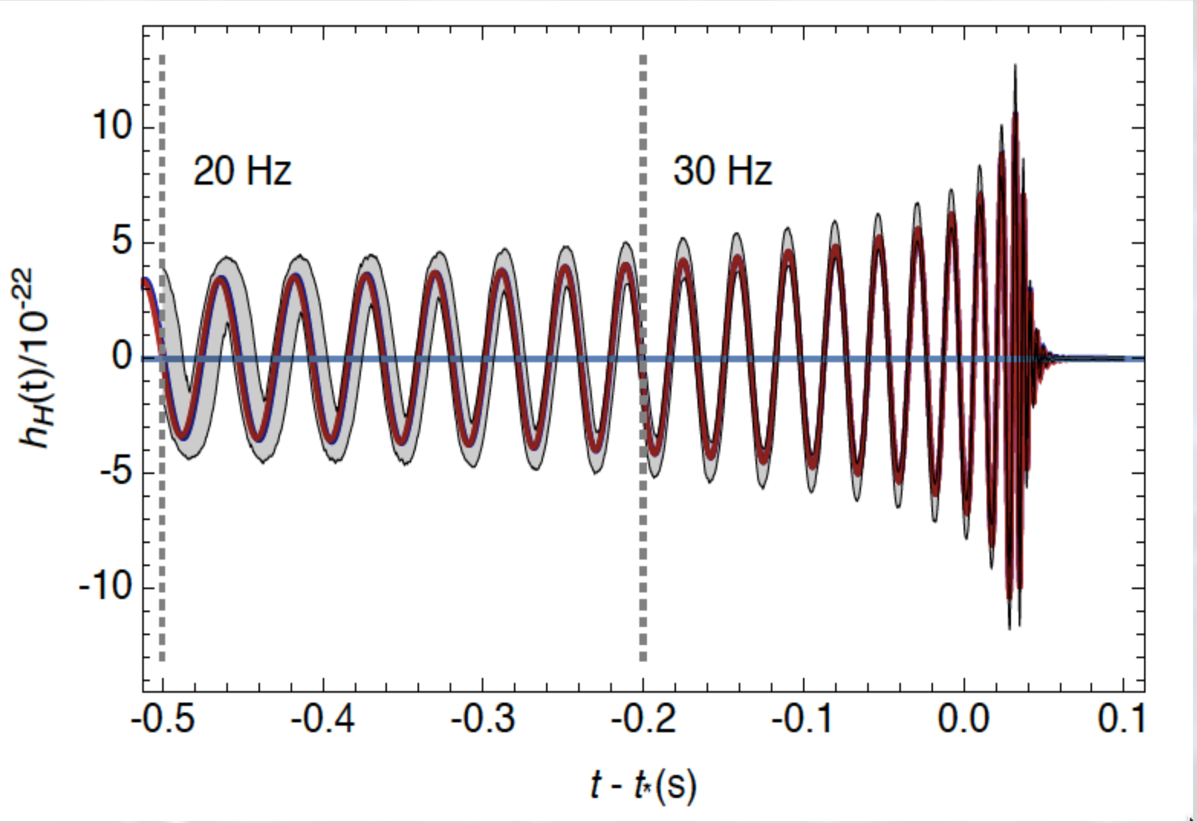


FIG. 3. Waveform polarizations h_+ (blue) and $h_\times$ (orange) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$ (equal to 0.2 s for a $20M_\odot$ BBH).

http://www.black-holes.org/waveforms

A. H. Mroué et al., Phys. Rev. Lett. 111, 241104 (2013)

Numerical Relativity Waveforms for GW150914



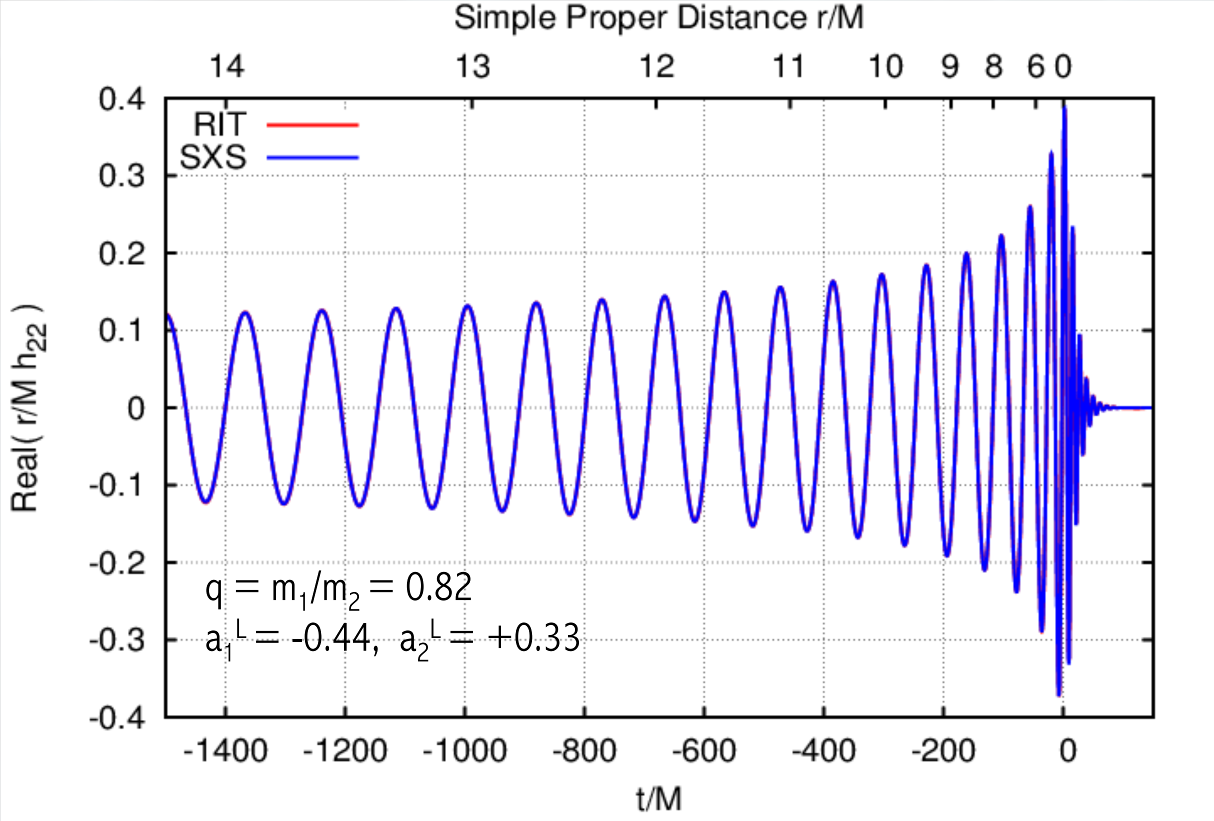
Directly comparing GW150914 data with thousands of NR simulations of BBH coalescence.

Predictions of NR simulations vs measured 90% probability that GW150914's strain is inside of the shaded region.

NR simulations (~1week) providing fast response to model GW150914.

LSC + NR teams (RIT, SXS, Georgia Tech, and the BAM team) arXiv:1606.01262

Numerical Relativity Waveforms for GW150914



Perfect agreement also among totally independent codes, and totally independent methods!

99.9% overlap among RIT and SXS NR waveforms used in the models for GW150914, Lovelace & Lousto +, 2016

See second row of Fig 1 of the GW150914 detection paper: Phys. Rev. Lett. **116**, 061102 (2016)

Completely Different and Independent NR methods!

	Moving-puncture	Excision
Initial data		
Formulation of Einstein constraint equations	Conformal method using Bowen-York solutions [19–21]	Conformal thin sandwich [20, 22]
Singularity treatment	puncture data [23]	quasi-equilibrium BH excision [24–26]
Numerical method	pseudo-spectral [27]	pseudo-spectral [28]
Achieving low orbital eccentricity	post-Newtonian inspiral [29]	iterative eccentricity removal [30, 31]
Evolution		
Formulation of Einstein evolution equations	BSSNOK [32–34]	Generalized harmonic & constraint damping [8, 35–37]
Gauge conditions	evolved lapse and shift [38–40]	Harmonic ($H^\mu = 0$) or evolved H^μ [41]
Singularity treatment	moving punctures [9, 10]	excision [42]
Outer boundary treatment	Sommerfeld	minimally–reflective, constraint–preserving [37, 43]
Discretization	high-order finite-differences [44, 45]	pseudo-spectral methods
Mesh-refinement	adaptive mesh refinement	domain decomposition [28, 41]
Codes		
Codes	BAM, Hahndol, LazEv, Lean, Llama, MayaKranc, UIUC	SpEC

Binary Black Hole Simulations – Numerical Relativity in "3+1"

- Spacetime sliced into 3-D t =constant hypersurfaces
- Einstein's eqns split into 2 sets:
  - Constraint equations (no time derivative)
  - Evolution equations
- Set (physical, but constrained) initial data at t = 0
- Evolve forward in time, from one slice to the next
- Solve $\geq$ 10 nonlinear, coupled PDEs

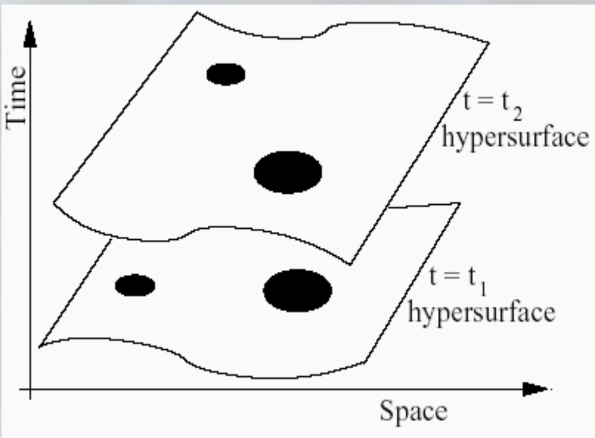
$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} = 8\pi \frac{G}{c^4} T_{ab}$$

$$R_{ab} = \sum_{c=1}^{4} R^{c}_{acb};\ \ R = g^{ab} R_{ab}$$

$$R^{c}_{acb} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{cd} + \Gamma^a_{ce}\Gamma^e_{bd} - \Gamma^a_{de}\Gamma^e_{bc} \leftarrow 1 \text{ Derivative}$$

$$\Gamma^a_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc}\right) \leftarrow 1 \text{ more Derivative}$$

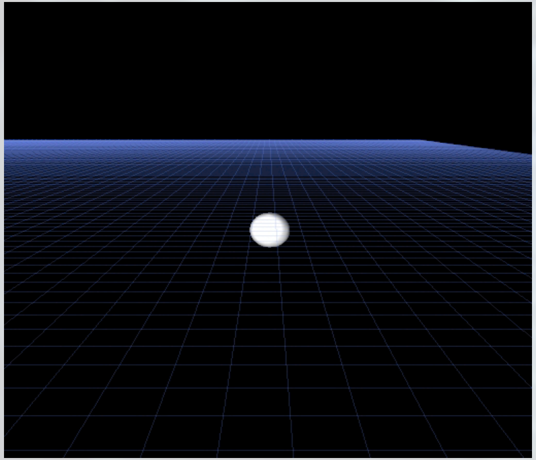
They change characters depending on the choice of variables, and coordinate system (gauge)

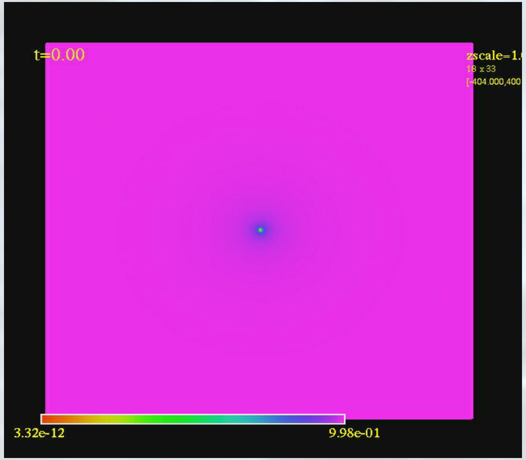


- Deal with coordinate or gauge conditions: relate coords on neighboring slices lapse α, shift vector β^i
- Deal with the BH singularities (excision vs puncture)
- Extract the physics: gravitational radiation quantities, final remnant parameters, etc

Simulations with Moving Punctures – Numerics

Simulations are evolved forward at discrete points in 3D space.
Time is advanced in discrete steps.
We need to determine several functions at every point in spacetime, describing how the distances between points change over time.





Simulation of high mass ratio BBH showing AMR grids adapted to different time and spatial scales; simulation on thousands of processors.
[Lousto & Zlochower, Phys. Rev. Lett. 2011]

Advanced numerical algorithms translates into more 300,000 lines of codes, that must scales well on thousands of processors.

Einsteintoolkit.org

ZILHÃO and LÖFFLER, Introduction to the Einstein toolkit, (2013)

Supercomputers

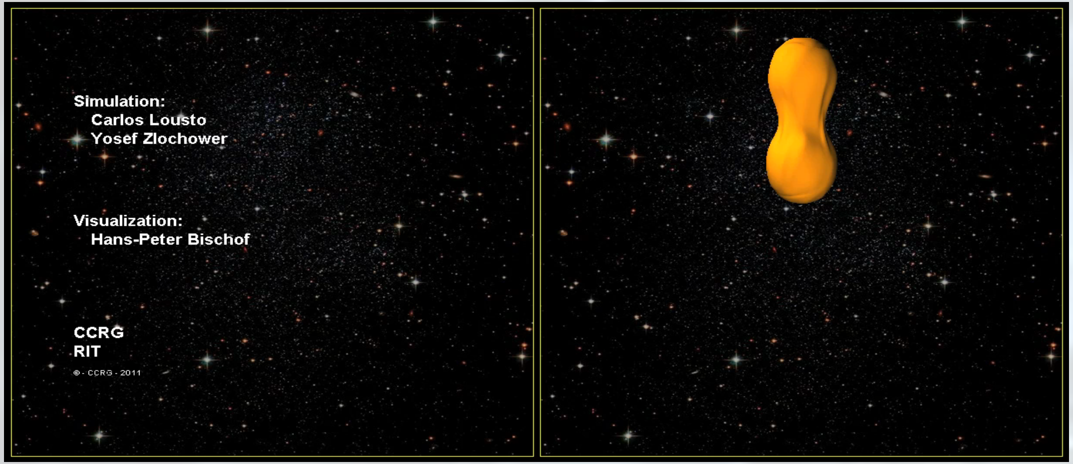
Supercomputers of thousands of processors are needed to accurate model these astrophysical phenomena

Vanilla simulations use tens to hundreds of millions of individual time steps, and tens of TB of online ram.

We need fast processors with peak petaflops performance (e.g. quadrillion operations per second), excellent interconnect, lots of memory per node to calculate our answer …

Gravitational-Wave Recoils

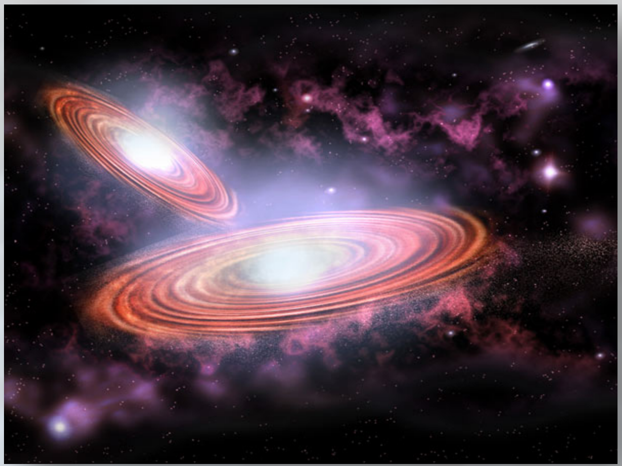
When black holes collide they can convert more than 10% of their total mass into very powerful bursts of gravitational waves which travel across the cosmos



This radiation is so intense that the black will recoil up 10,000,000 mph. If the recoil is large the black hole can escape the galaxy.

Black Holes Orbital Dynamics and Kick (Left) and Gravitational Radiation Power (Right)
Campanelli et al Phys Rev Lett, 2007, González et al Phys Rev Lett 2007,
Lousto & Zlochower, Phys Rev Lett, 2013

Spin Dynamics in Binary Black Holes



When the BH spins are aligned with the orbital angular the merger is delayed (orbital hang-up effect) due to spin-orbit coupling.

Campanelli+, Phys Rev Lett, 2006;
Lovelace+, Phys. Rev. D, 2011

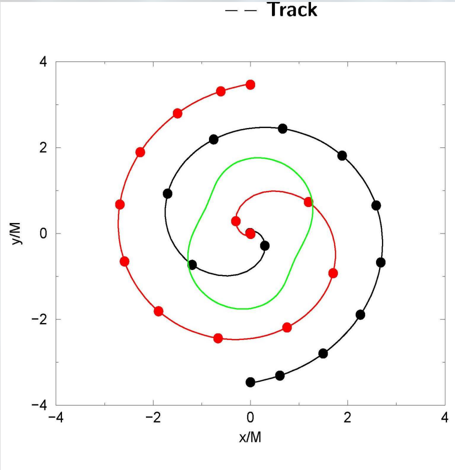


Figure 4: Puncture tracks for the −− configuration.

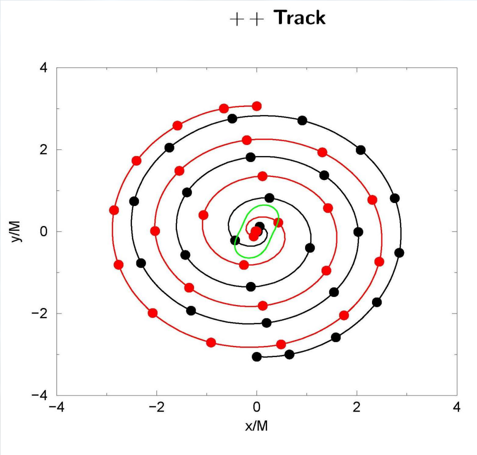


Figure 6: Puncture tracks for the ++ configuration.

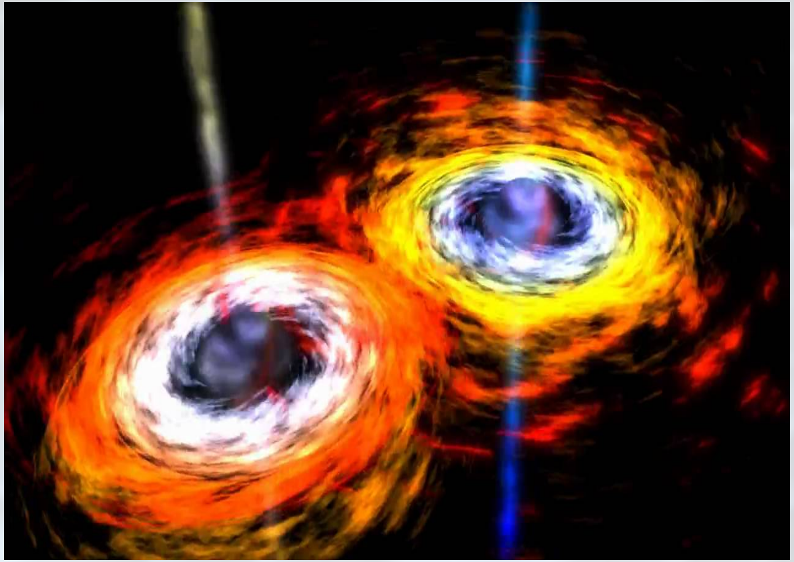
If the spins are not aligned then the spin of one of the BHs can reverse completely (spin flip-flop) during the inspiral and merger with consequences for accretion disk

Lousto & Healy, , Phys. Rev. Lett., 201

Can Supermassive Black Hole Binaries be Bright?

Supermassive black holes in gas rich environments can produce very energetic emissions of electromagnetic radiation (optical, radio, etc), due to accreting gas in strong dynamical gravity.

They can also launch powerful **jets** that reach out to thousands of light years from the source.

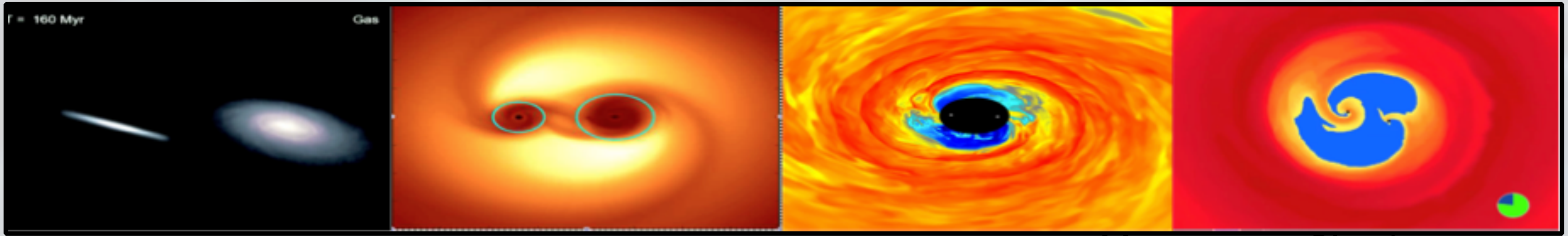


Intensive, high-cadence, high-resolution astronomical surveys,
e.g. **Large Synoptic Survey Telescope (LSST)**, could observe these mergers, providing electromagnetic counterparts to future gravitational-wave observations, e.g. LISA.

Modeling Binary Black Holes in a Gaseous Environment

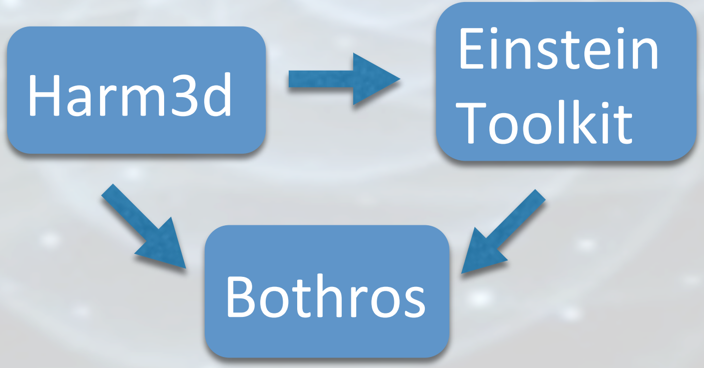
Need accurate calculations of the sources temporal variability and energy spectrum, and scales range 10 order of magnitude (from 10 Kpc to 10^{-6} pc), and the parameter space is huge!

Hopkins, Hernquist, Springer et al | Farris++2013 | Noble++2012 | Farris++2011

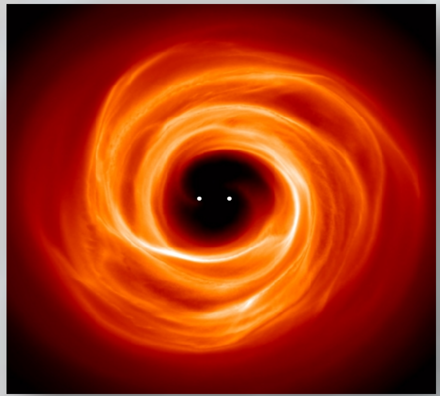


Galaxy Mergers | Binary Formation | Inspiral | Merger | Ringdown

- Magnetohydrodynamics (MHD)
- General Relativity BBH Inspiral and Merger
- Ray Tracing
- Radiative Transfer
- Multi-species Thermodynamics

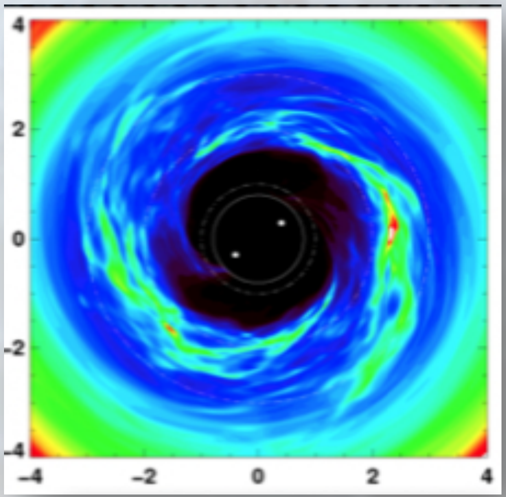


Accretion Disk Dynamics and Binary Black Holes

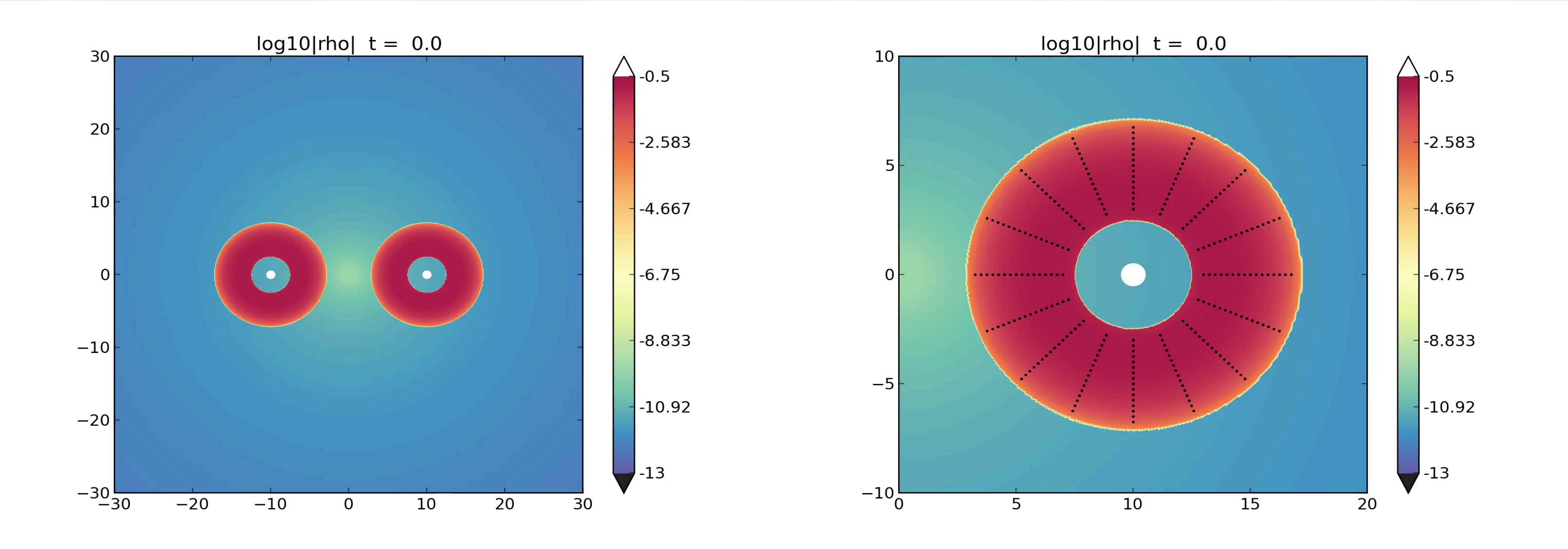


2D hydro simulations find that binary tidal torqueing on the gas evacuate region near BHs [Macfadyen & Milosavljevic 2008; Cuadra+ 2009]

But in radiatively efficient magneto-hydrodynamics (MHD) simulations, **accretion is not reduced by presence of binary** [Shi & Krolik 2015, Farris+ 2014], but it continues until very close to merger [Noble ++ 2012]



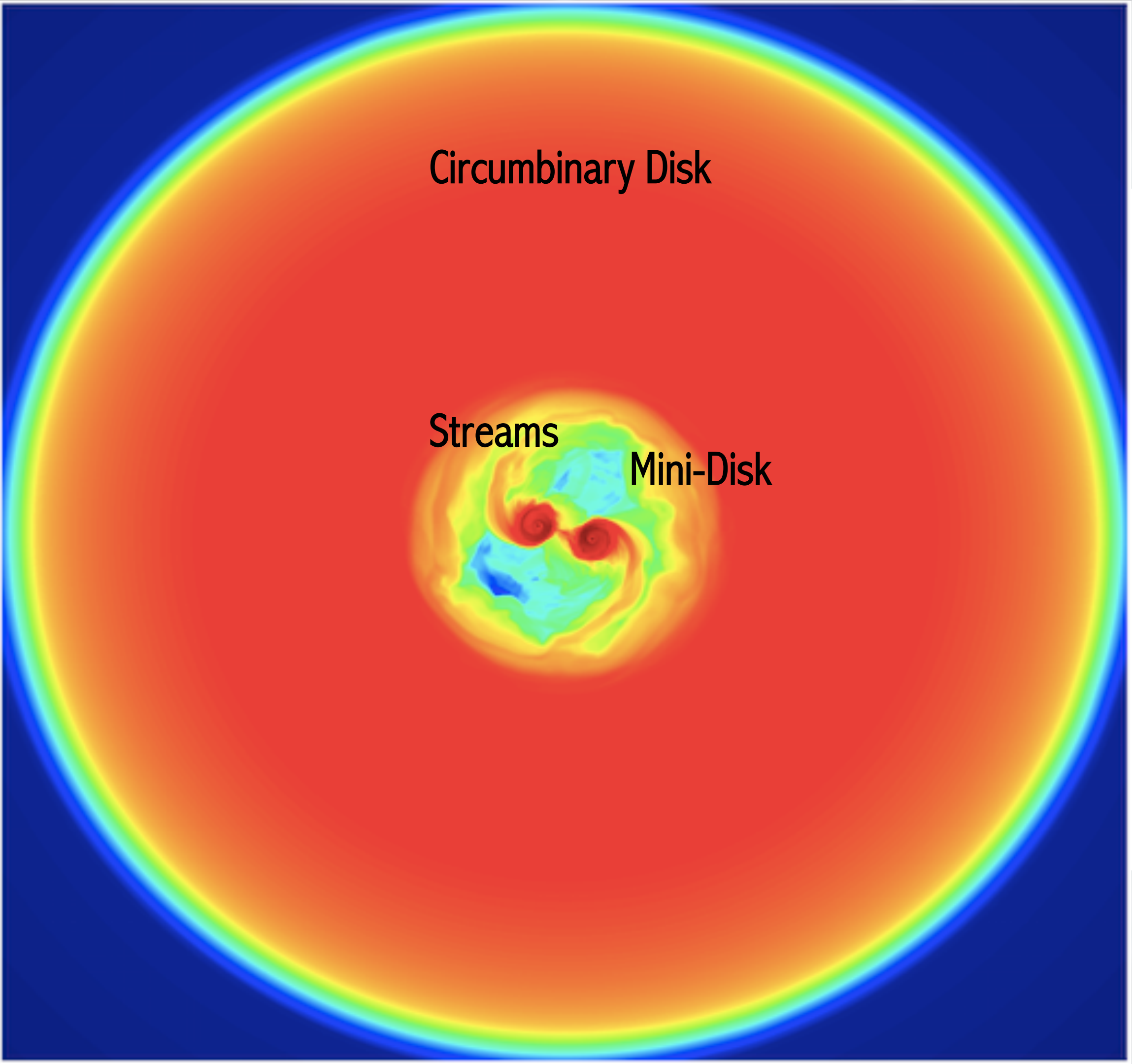
Following the Gas Dynamics Close to each Black Hole

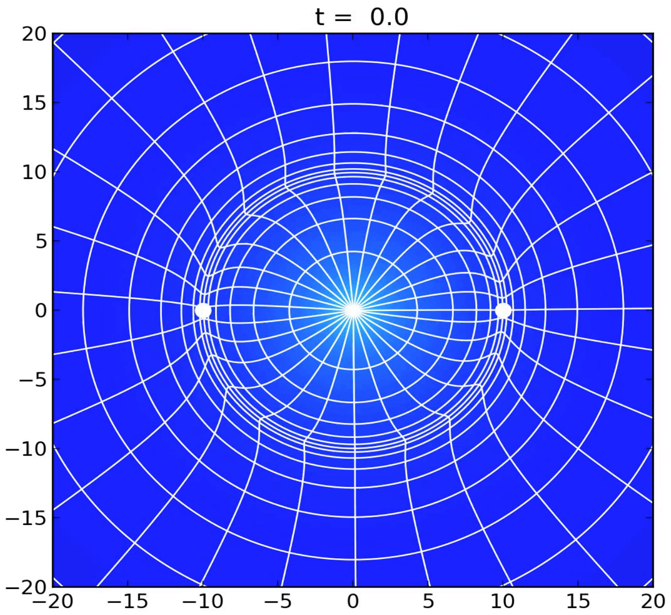


Mini-Disks are stable through late stages of the binary black hole inspiral, and may serve as a key component of the EM signature o supermassive black hole merger

[Bowen, Campanelli, Noble and Krolik 2016]

Following the Gas Dynamics Close to each Black Hole

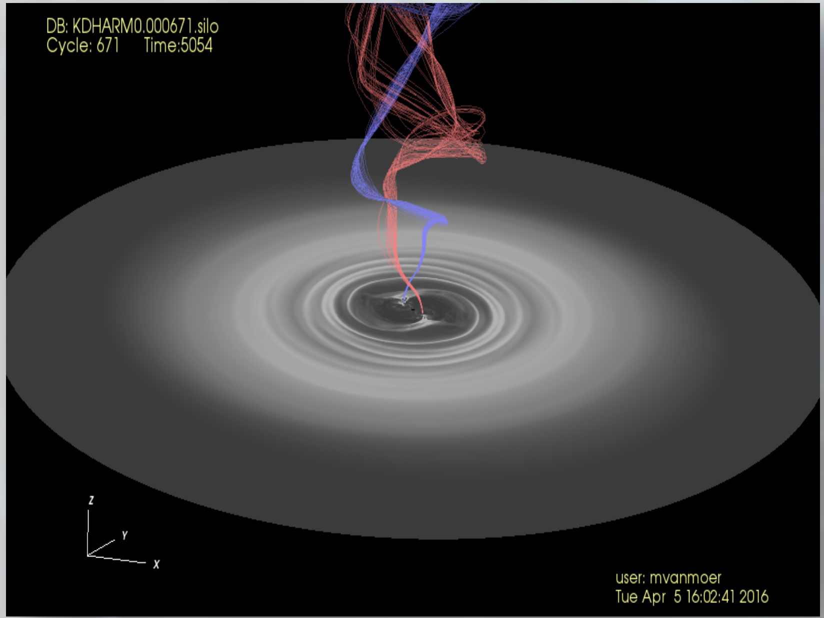




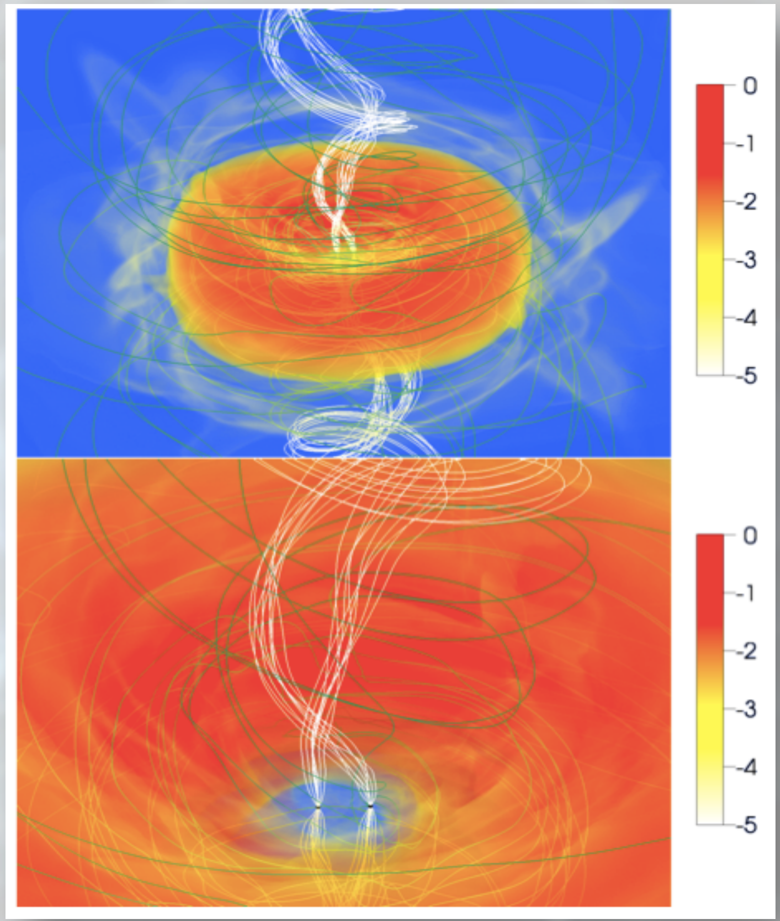
Warped Grids to concentrate grid points near BHs [Zilhão & Noble, 2014]

Full GR-MHD Simulations ...

Exploratory work in the very dynamical regions close to the merging BHs already suggest very exciting results.



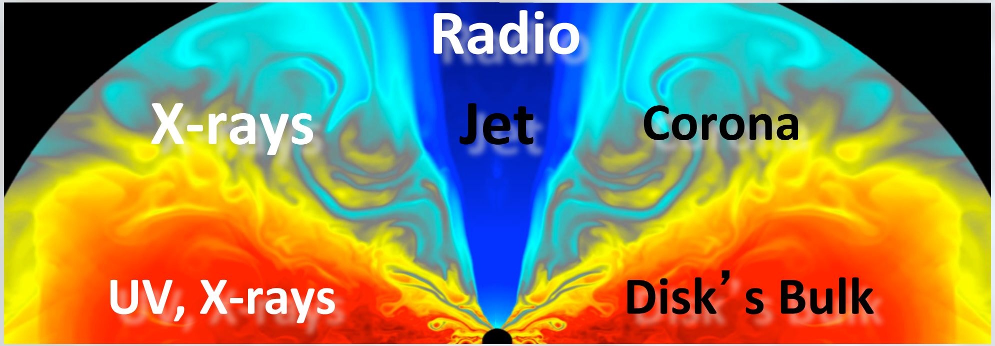
Noble, Bowen, Campanelli and Krolik, 2016



Gold++ 2013

General Relativistic MHD Framework

Finite-volume/finite-difference code, Harm3D, to solve magneto-hydrodynamics (MHD) for thin (radiatively efficient/cooled) accretion disks in dynamical spacetimes [Noble+ 2009]



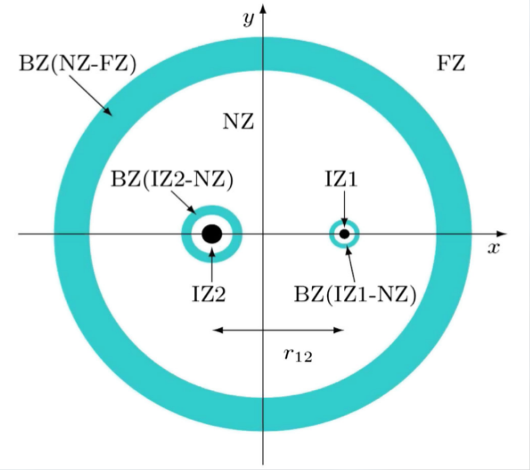
Matter evolves through conservation of mass, energy and momentum and Maxwell's equations

$$\frac{\partial}{\partial t}\sqrt{-g}\begin{bmatrix}\rho u^t \\ T^t{}_t + \rho u^t \\ T^t{}_j \\ B^k\end{bmatrix} + \frac{\partial}{\partial x^i}\sqrt{-g}\begin{bmatrix}\rho u^i \\ T^i{}_t + \rho u^i \\ T^i{}_j \\ \left(b^i u^k - b^k u^i\right)\end{bmatrix} = \sqrt{-g}\begin{bmatrix}0 \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{t\kappa} - \mathcal{F}_t \\ T^\kappa{}_\lambda \Gamma^\lambda{}_{j\kappa} - \mathcal{F}_j \\ 0\end{bmatrix}$$

$$T_{\mu\nu} = (\rho + u + p + 2p_m)\, u_\mu u_\nu + (p + p_m)\, g_{\mu\nu} - b_\mu b_\nu$$

Mass Density; Internal Energy Density; Gas Pressure; Fluid's 4-velocity; Magnetic Pressure; Magnetic 4-vector; Radiative Energy & Momentum Loss

Binary black hole spacetime and dynamics treated analytically during PN inspiral



Strong Field Gravity Astrophysics

Colliding compact objects, including **black holes** and **neutron stars**, **supernova explosions** are the ideal sources for testing strong gravity as they all generate very powerful bursts of gravitational waves.

They are often accompanied by powerful bursts electromagnetic (EM) radiation, and neutrinos, and jets, making then the **ideal astrophysics laboratory for understanding matter and electromagnetism in the most extreme conditions.**

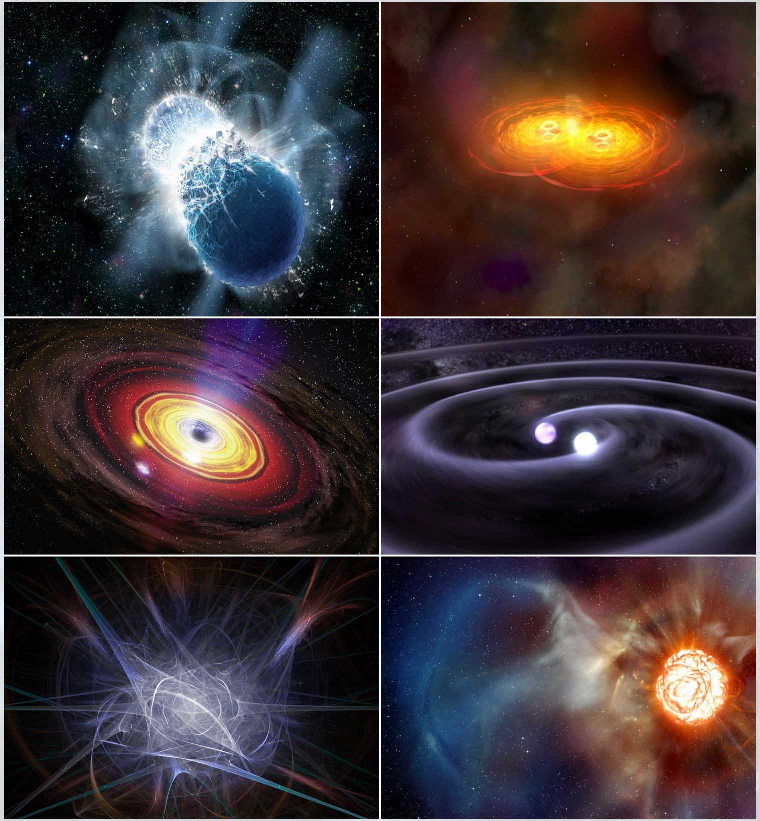
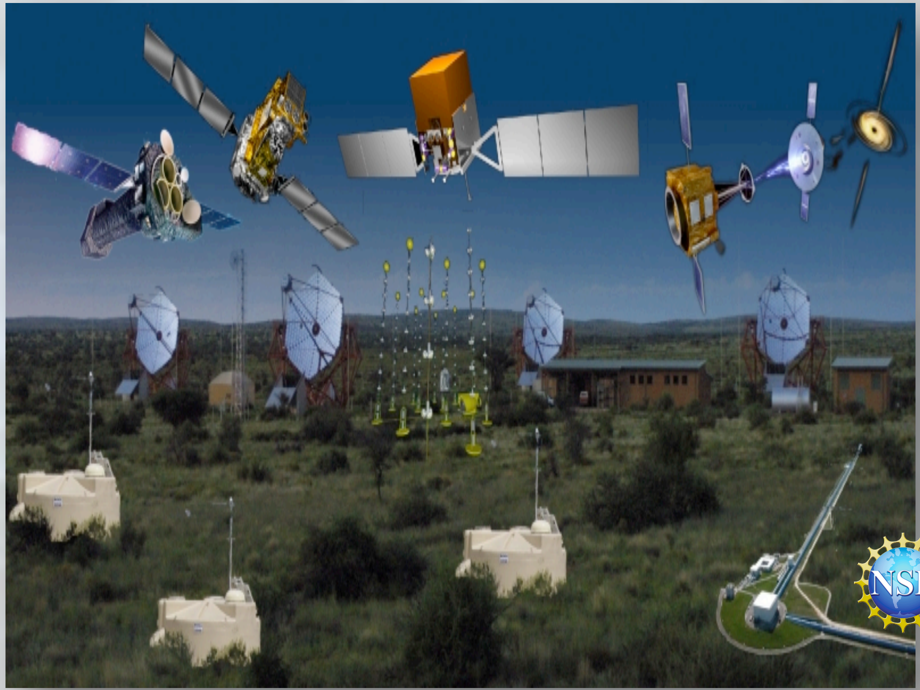


Image credits: Shane Larsen

Multi-messenger Astronomy

Combining electromagnetic observations (gamma-rays, x-rays, optical, radio) of these phenomena with gravitational-wave observations is an emerging new field of astronomy

- Independent sky localization
- Better understanding of distance vs. redshift relationship, with implications for cosmology
- Better understanding matter and electromagnetism in the most extreme conditions.



Concluding Remarks

Advanced LIGO, just made its first detection of a binary black hole merger's signal which matches with the numerical relativity simulations just beautifully!

Some new amazing predictions about the physics and astrophysics of binary black hole mergers, and other compact objects, will be made in the near future as we move forward with multi-messenger astronomy!

Our way to look at the Universe has totally changed!

"It's black, and it looks like a hole. I'd say it's a black hole."

Thank You!

Center for Computational Relativity and Gravitation

National Science Foundation