ACTIVITY 4

Matter Waves

Goal
We will use the results of the previous experiments and establish quantitative concepts for electron waves.

In the previous activity, we saw that in certain experiments electrons produce patterns attributed to waves. From this observation, we concluded that electrons have wave-like properties. You also concluded that as the energy of electrons increases their wavelength decreases.

Electrons are a form of matter, so these waves are called matter waves. In this activity, we will relate quantitative features of matter waves with familiar, measurable physical quantities such as energy, mass, and momentum. We will apply these relationships to forms of matter other than electrons, and see how these results can be applied to the electron microscope.

Louis de Broglie was the first person to establish an equation for the relationship between an electron's momentum and its wavelength. He concluded that:

$$Wavelength = \frac{Planck's\ \ Constant}{Momentum} \qquad \lambda = \frac{h}{p}$$

where *h* is a number called Planck's constant (named after Max Planck) and is equal to 6.63×10^{-34} J × s or 4.14×10^{-15} eV × s.

When we observe electron diffraction, the electrons' kinetic energy is easier to measure than their momentum, so we write the de Broglie wavelength as

$$Wavelength = \sqrt{\frac{(Planck's\ \ Constant)^2}{2 \times Mass \times Kinetic\ Energy}} \quad \text{or} \quad \lambda = \sqrt{\frac{h^2}{2mKE}}$$

This equation is consistent with our results in the previous activity — as the energy increases the wavelength decreases.

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While we have learned about this equation using electrons, it can be used for any type of matter. So, you might wonder why we do not see wave effects for large objects.

Start the *Double Slit* program. To begin to understand and experiment with objects with masses greater than that of an electron, run the simulation of the experiment for electrons. Throughout this experiment keep the same energy.

Next choose *pions* and repeat the process. Repeat for *neutrons*, and finally *protons*. (A pion has a mass 270 times that of the electron while the neutron and proton masses are about 2,000 times that of the electron.) After doing these experiments, feel free to vary any of the parameters except the energy. Then, answer the following questions. Compare the diffraction patterns made using the other types of matter with that of the electron.

? How do the distances between dark areas change with the mass?

? How does the de Broglie wavelength change?

? How do the patterns for protons compare to the patterns for neutrons?

? Use de Broglie’s hypothesis to explain this similarity or difference.

Now, investigate how the patterns change as the separation between slits changes. Pick one particle and one energy; change the slit separation to answer the following.

? How does the pattern change as the slit separation increases?

For patterns to be formed, the separation between the slits must be comparable to the wavelength of the waves passing through them. Thus, as your separation became very large, the pattern was not easy to see.

While wave behavior is exhibited by electrons, pions, neutrons and protons; we do not observe similar behavior for large objects such as gnats or humans. As an example, we will consider why diffraction doesn't cause a gnat to look like several gnats as it flies through window blinds. Suppose the gnat's mass is .001 kg, and its speed is 0.10 m/s.

? What is the gnat's momentum?

? What is the gnat's de Broglie wavelength?

? Approximately what would the spacing between the window blinds have to be for the gnat to create a pattern as it flew through? Why?

? Is it necessary for a gnat to worry about creating a pattern as it flies through the blinds? Why?

? How would a human being's de Broglie wavelength compare to that of a gnat? Why?

Because we know the values of Planck's Constant and the electron's mass, we can use them to simplify the equation to apply only to electrons as

$$Wavelength = \sqrt{\frac{1.5\ nm^2 \cdot eV}{KineticEnergy}} \quad \text{or} \quad \lambda = \sqrt{\frac{1.5\ nm^2 \cdot eV}{KE}} \quad \text{[ELECTRONS ONLY]}$$

where the kinetic energy has been measured in electron volts (eV). The result, l, is the electron's wavelength in units of nanometers. This equation works only for electrons, so use it carefully.

? How does the relationship between energy and wavelength in this formula compare with the relationship that you observed in Activity 3?

A valid question to ask is: "What is waving with these matter waves?" Unfortunately, the answer is not an easy one. We never observe a matter wave directly; we only see results that can be explained by them. The matter wave is an abstraction that allows us to explain observations. In the next activity, we will look at what information is contained in waves of matter. We will examine the features of matter waves that describe simple properties such as the location of an object.

An Application

Because electrons behave as waves, they can be used to "illuminate" objects in a manner similar to light. An electron microscope is an instrument that takes advantage of this situation. Electrons are given energy by accelerating them in a manner similar to the way a TV tube works. Then, using magnetic fields, they are directed at an object of interest. The electrons are focused to illuminate the object, and then to form the image of that object. A schematic diagram is shown in Figure 4-1. This system can be used to look closely at very small objects.

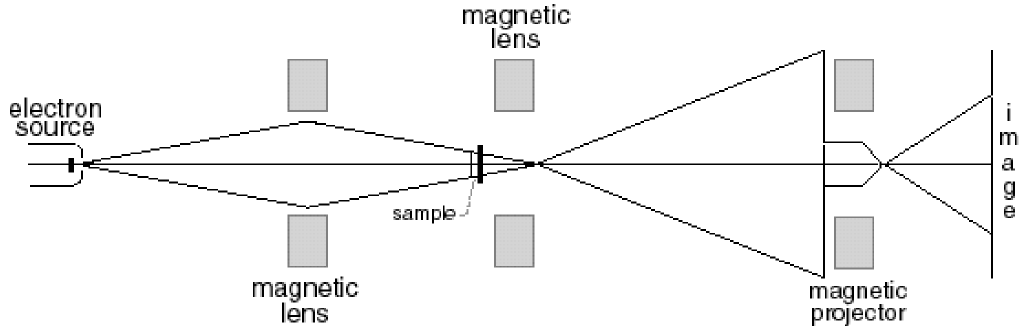

Figure 4-1: Schematic diagram of electron microscope.

The wavelengths of the electrons are related to their kinetic energies. In electron microscopes, wavelengths as much as 100000 times smaller than those of visible light can be achieved. With such small wavelengths, electron microscopes can reveal features that are as small as 0.000000001 meters (1 nm). Below are some electron microscope pictures.

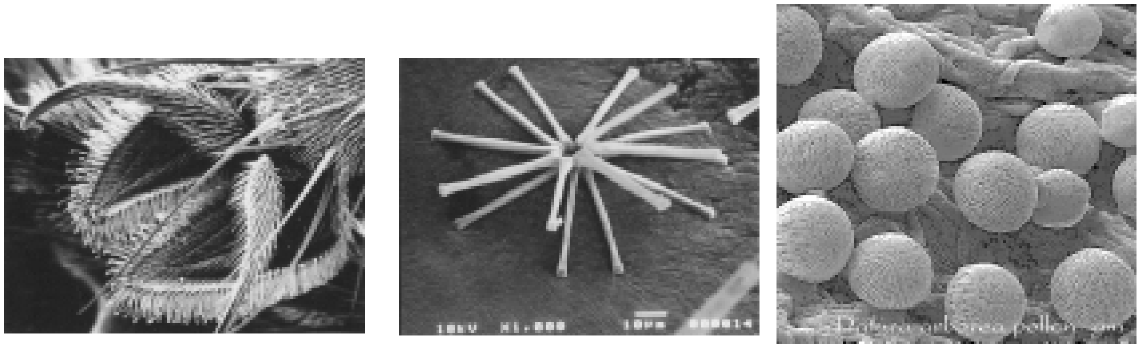

Figure 4-2: Electron microscope images of: (a) the foot of a housefly; (b) a diatom; and (c) pollen.

Homework

A situation where matter waves could become important is the Star Trek transporter. We are not sure how a transporter would really work, but for the purposes of this activity, let us suppose that it decomposes a person into his or her component atoms. Then, it sends the atoms to a new location where the person is reconstructed.

Consider transporting Captain Janeway of the Voyager by such a method. She wishes to reach her new location quickly, so her atoms are sent out of the ship at 10% of the speed of light (3×10^7 m/s). Assume that her atoms have a mass of 10^{-26} kg

? What is the de Broglie wavelength of each matter wave?

Each atom must be transmitted through the titanium hull of the starship. The titanium can be considered as a large number of slits separated by 1 nm.

? Must the designers of the transporter be concerned about diffraction effects as the captain's atoms are beamed through the hull of the Enterprise? Why or why not?

? Would this effect make a good premise for a Star Trek movie? Why or why not?