Name Partners Date

Energy Diagrams I

Goal
Changes in energy are a good way to describe an object's motion. Here you will construct energy diagrams for a toy car and learn how these diagrams can be useful. This technique will prepare you for similar uses of energy diagrams in quantum physics.

Introduction

In quantum mechanics the concept of energy is frequently used to help us describe events and motion. Instead of describing changes in motion in terms of forces and accelerations, we use changes in energy. A convenient way to describe changes in energy is to sketch a graph of the value of an object's energy at various locations. When we create such a graph for energy, we call it an *energy diagram*.

Using energy to describe motion is somewhat different from the typical approach. However, it is a very powerful technique that can be used for both small and large objects. To prepare you for learning about atoms and other small objects you will first describe the motion of a toy car using energy diagrams. You will see that you will be able to describe this motion without actually seeing that object or its motion. All you need to know is the energy.

An example of a *potential energy diagram* is shown in Figure 1. On the horizontal axis is the location of the car while the vertical axis shows the value of the potential energy at each location. We shall see that we can learn a lot about the car's motion from this diagram.

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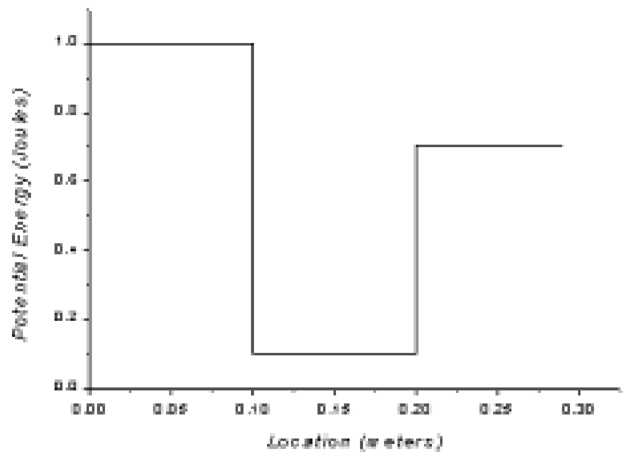


Figure 1: A potential energy diagram for a toy car.

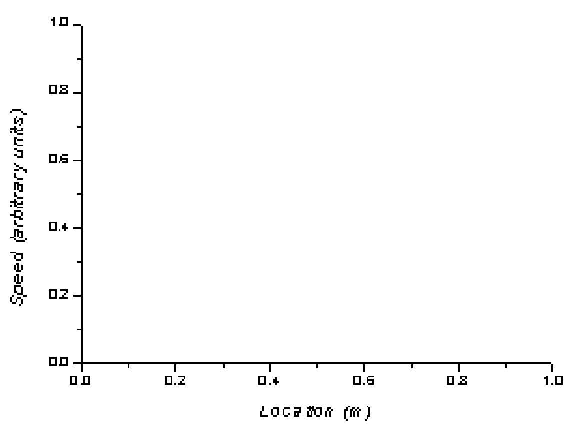
The potential energy can arise from a variety of interactions including elastic, gravitational, chemical, etc. In this tutorial, the potential energy changes are generated with the help of magnets. However, the interactions between atoms and nuclei are of different origin. Therefore, the comparison with the atom that you will attempt later on will only be concerned with the shape of the potential energy curve, not with the origin of the interactions.

A. Energy Diagram for No Interactions

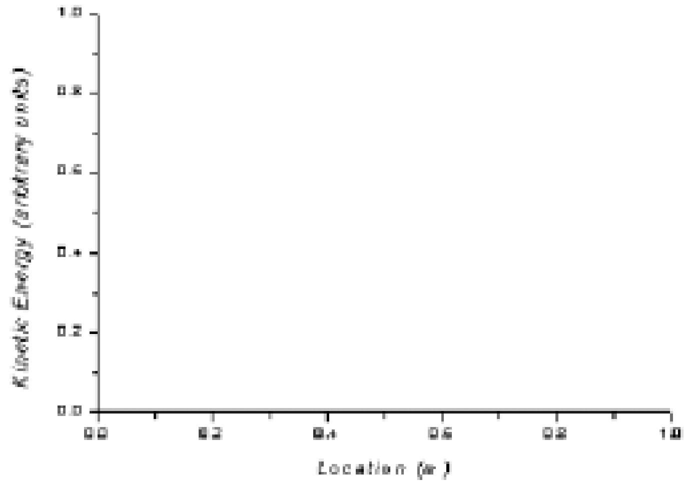
In this activity you will arrange magnets along a track and observe the interaction between these magnets and a magnet on the car. By changing configurations of the magnets along the track you will obtain potential energy diagrams of various shapes. You will begin with the magnets in *attractive* mode.

In this activity we will frequently make approximate sketches of the energy of the toy car. As a first example, consider what happens when you push the car along the track with no magnets near the track or on the car. Try it now by giving the car a gentle push.

A-1. Use your observation of the motion and sketch a graph of speed vs. location. Your graph should be just a rough sketch. We do not measure the values of the speed, so we can only approximate the shape of the graph.



A-2. Now, sketch a rough graph of the kinetic energy vs. location. (Hint: The kinetic energy is proportional to speed^2.)



Again, we have not measured the kinetic energy, so we can not put values on the vertical axis, but we can learn a lot from the shape of the curve. For example, your curve probably shows that the kinetic energy decreases with distance. This result tells us that the car's energy decreases as it moves along the track. That is not a surprise, because the car has internal friction as well as friction with the track. Your graph gives you an idea about the rate at which the friction changes the speed.

The friction between the wheels of the car and the track is relatively high. If we were to account for that friction, it would have made the energy diagrams complicated. However, for the purposes of our qualitative analysis we will try to think about what would have happened if the friction acting on the car were very small. For example, the decrease in kinetic energy in your graph is caused by energy going into work against friction. If friction were not present, the sketch of kinetic energy vs. location would be a straight line as shown in Figure 2.

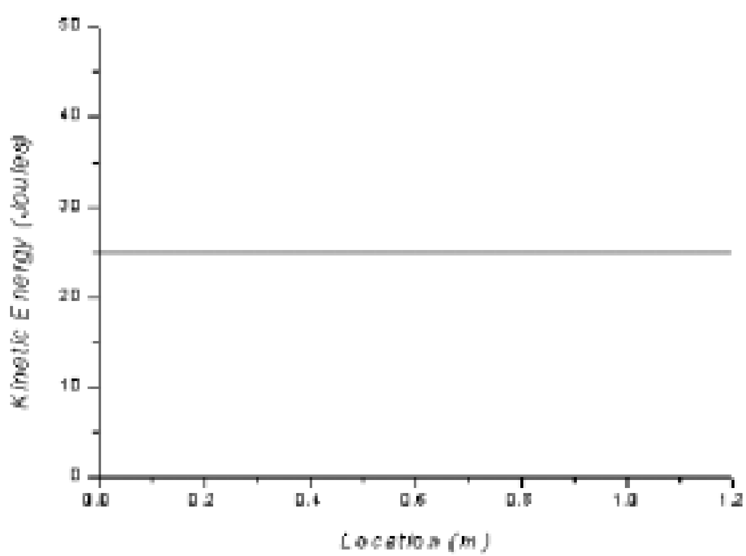


Figure 2: A graph of kinetic energy vs. location for a car that loses none of its energy to friction.

Consider another example - the graph of the total energy of the car when it is moving on a level track. Because we ignore the effect of friction, the total energy (total energy = potential energy + kinetic energy) will not change. Thus, the graph will look like Figure 3.

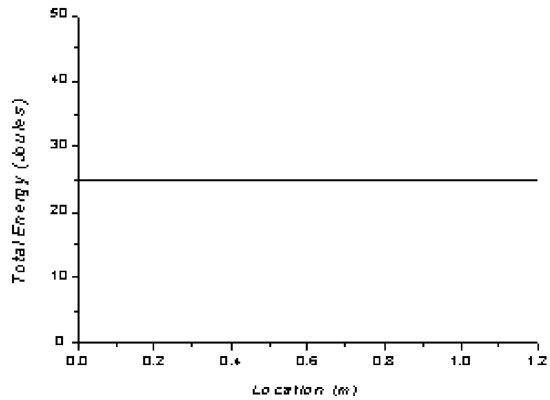


Figure 3: The total energy vs. location for a car that loses none of its energy to friction.

B. Exploring Interactions Between Attractive Magnets

Now we will add interactions to our car. The interactions will occur between magnets on the car and magnets along the track.

The diagrams in Figures 4 and 5 show you how to set up the experiment:

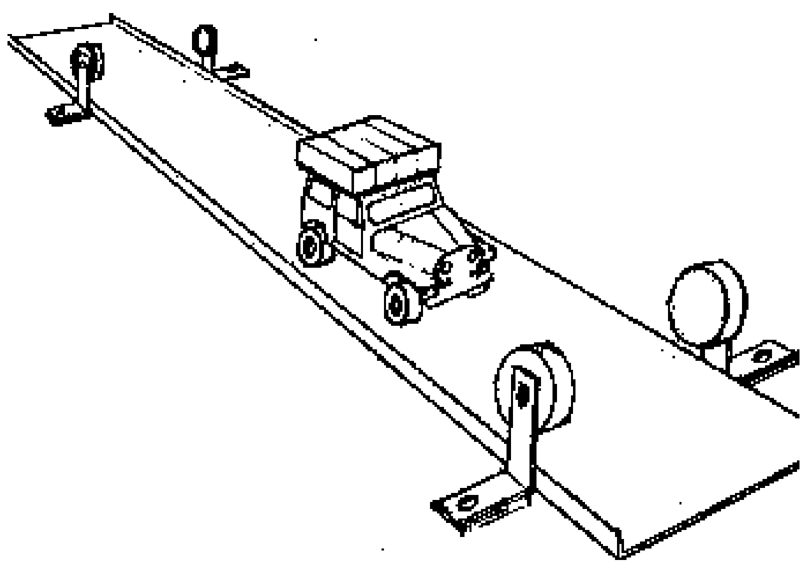


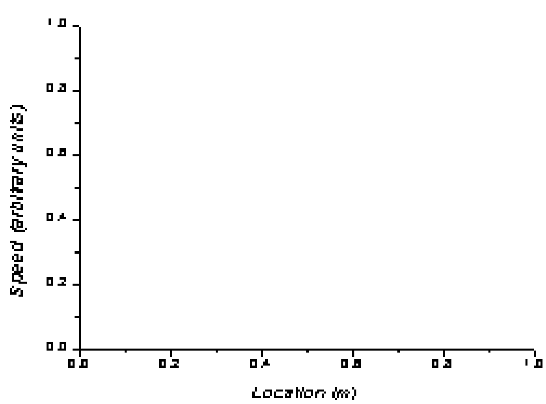
Figure 4: The experiment arrangement for the car interacting with magnets.

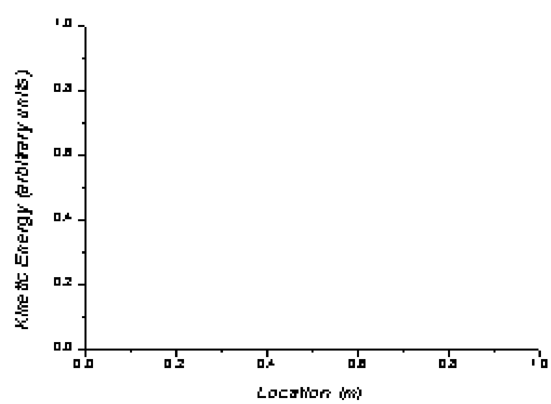
Figure 5: A car with a magnet and a magnet holder.

Push the car and watch carefully as it goes through the magnets. If the car does not go beyond the magnets, try again with a stronger push.

B-1. How did the speed of the car change as it approached and then passed the magnets? Write down your observations about all speed changes.

B-2. In the space below, draw a rough sketch of the speed changes vs. location and then kinetic energy vs. location.





B-3. Indicate the location of the magnets on your diagram.

The graphs for the speed and kinetic energy have a large change in the neighborhood of the magnet. However, the diagrams should also show an overall decrease in value as the cars move along the track. As before, this decrease is due to interactions involving friction.

For the car with no magnetic interactions you had kinetic energy diagrams such as those in Figure 6.

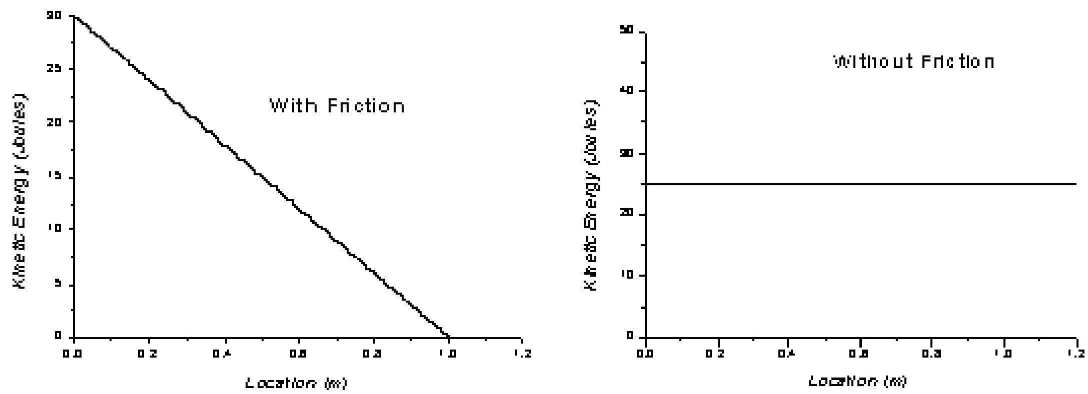
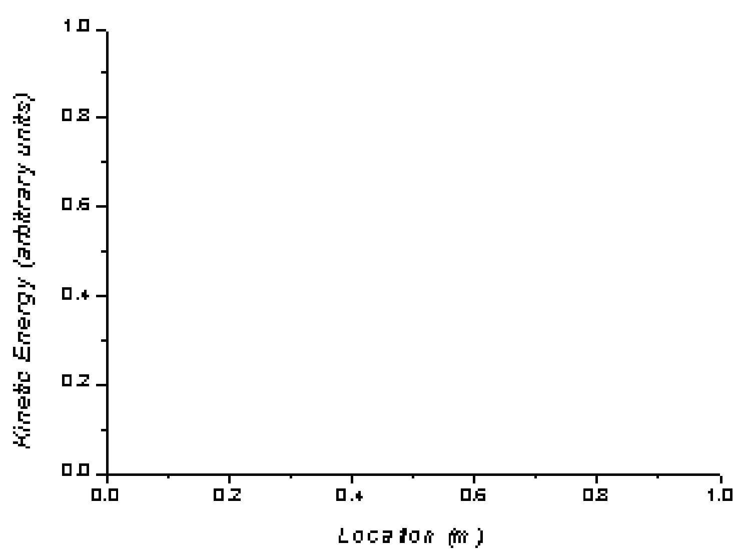


Figure 6: Kinetic energy diagrams for a car on a level track with friction (left) and no friction (right).

The change from "without friction" to "with friction" in Figure 6 can help us think about how other graphs would look if friction were not present. Imagine the kinetic energy diagram that you drew for the car interacting with a pair of magnets.

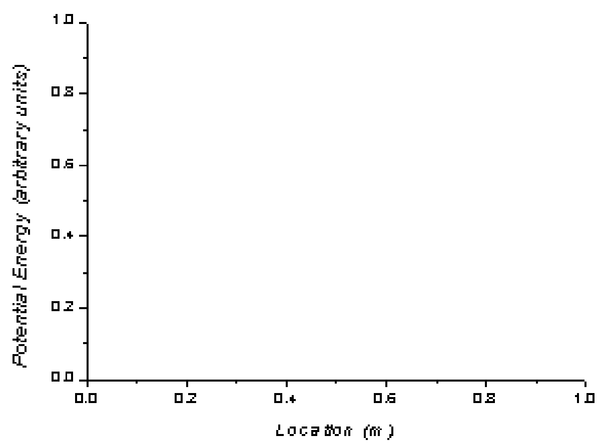
B-4. Draw it in the space below as if friction were not present.



B-5. On the “no friction” kinetic energy diagram in Figure 6, draw a line to represent the total energy of the car. Include the interactions with the magnets. Hint: Far from the magnets, the total energy equals the kinetic energy of the car.

The change in kinetic energy near the magnets shows that the kinetic energy becomes greater than the total energy. However, we know that energy is conserved. Thus, another form of energy must also change in the region near the magnets to keep the total energy constant. This other energy is the potential energy resulting from the magnetic interactions. We can use information about the kinetic and total energy of the car to construct a potential energy diagram of the car-magnet system. To construct a graph of potential energy vs. location, use potential energy = total energy - kinetic energy. In the present case we must approximate because we do not have exact values.

B-6. Sketch the potential energy diagram of the car by subtracting it from the kinetic energy diagram.



To maintain conservation of energy the potential energy must be negative in the region near the magnet. In fact, the shapes of the potential and kinetic energy diagrams turned out to be identical, although inverted. If we add the dip of the potential energy and the hump of the kinetic energy together, they will cancel out. The resulting diagram will be a straight line. This line represents the total energy of the car that does not change if friction is not present.

The graph of potential energy vs. location that you created is an example of a *potential energy diagram*. These diagrams can be useful in describing motion for all types of objects. Once you have the potential energy diagram for a situation, you can describe the motion of an object.

The usual method of using potential energy diagrams is to:

- start with the physical situation
- use the physics to draw the potential energy diagram
- describe the motion

In this way you can describe the motion even when you have never seen it. We can also describe other features in addition to speed and acceleration. In later activities we will look at some of those features.

The potential energy diagram provides an alternative way to describe motion of objects such as cars. They can also be used for analyzing the motion of very small objects — ones we cannot see. Thus, we need to first understand their value for objects we can see before using them with objects that are too small to see.

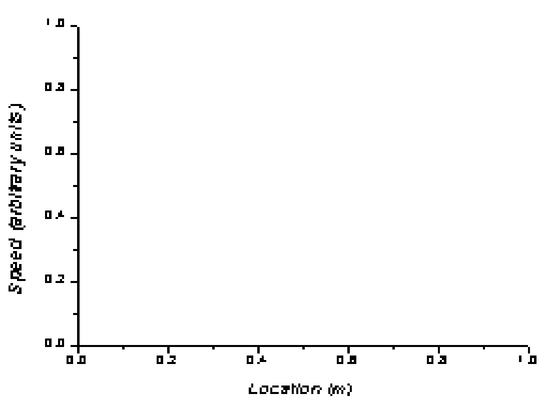
The experimental arrangement in this activity is similar to the previous one. The major difference is that we will arrange the magnets so that the ones along the track repel the one on the car. Again, we will try to imagine what the situation would be if the friction between the wheels of the car and the track was extremely small.

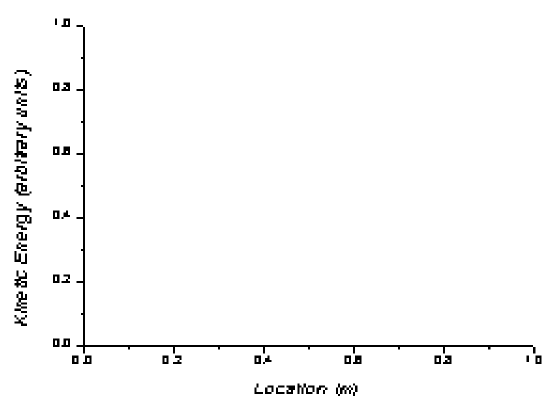
C. Energy Diagrams for Repulsive Situations

Use the same setup as in the previous section, but arrange the magnets so that the magnets along the track repel the magnets on the car. Push the car and watch carefully as it goes through the magnets. In the first observations we wish to consider a situation in which the car has enough energy to go past the magnets and continue along the track. If the car does not go beyond the magnets, try again with a stronger push.

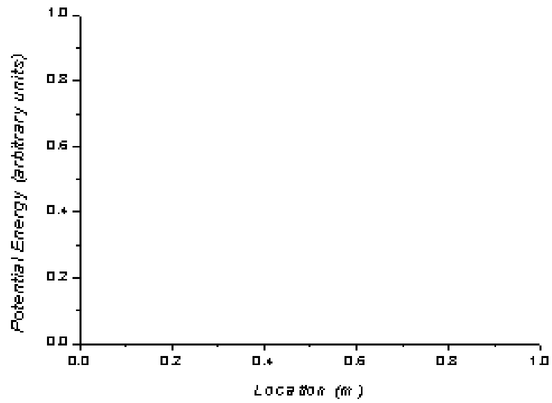
C-1. How did the speed of the car change as it went through the magnets? Record your observations below.

C-2. As you did before, plot *approximate* graphs of speed vs. location and kinetic energy vs. location. Mark the position of the magnets on the location axis.



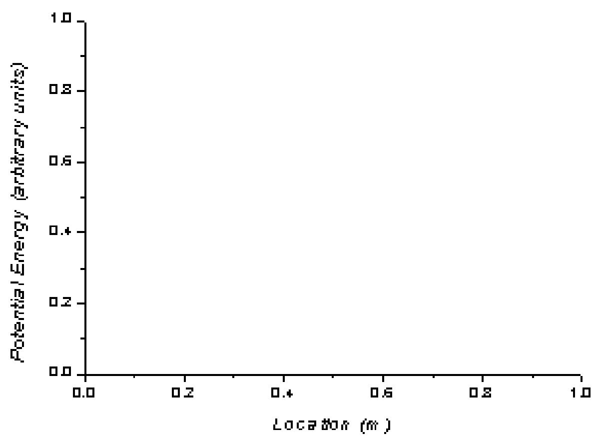


C-3. Now try to imagine the total energy with very low friction. Draw a line which represents the total energy of the car if no friction were present.



C-4. Add to the graph above the kinetic energy as if there were no friction.

C-5. Sketch below the potential energy diagram of the car by applying the conservation of energy.



C-6 How is this potential energy diagram different from the ones for the attractive situation?

C-7. Explain the reason for this difference in terms of how the energy of the car changes.

D. Simulating Energy Diagrams

So far we have tried to imagine the energy diagrams if friction could be removed. Because we cannot magically remove the friction from the toy cars, you will use a computer program to explore the frictionless case further.

Start the *Energy Diagrams Explorer* program. With this program you can place pairs of magnets along the track and give the car a push. You can also change the coefficient of friction. (A coefficient of friction equal to zero means no friction.)

D-1. Set up a computer version of the experiment that you just completed. Try it with a small amount of friction and describe your results below.

D-2. Now, set the coefficient of friction to zero. How do the results change?

D-3. Repeat this process for the experiment using repulsion. Describe your results below. If any of the results surprise you, discuss them with your instructor.

To check your partners' understanding of these ideas try a little game. While they are not looking at the computer screen, set up a configuration of magnets and a coefficient of friction. Run the car along the track to get the energy diagrams. Then cover the part of the screen that shows the car and magnets. Your partner(s) should look at the energy diagram and tell you

- the location(s) of the magnets,
- if each set is repulsive or attractive,
- the level of friction (zero, low, high), and
- how the speed changes as the cart moves along the track.

Don't make it too difficult. Your partners should set up a situation for you also. If any of the results surprise you, discuss them with your instructor.

E. Turning Points: Car Repelled By a Pair of Magnets

E-1. Based on what you have learned so far, how would you arrange the magnets (as attractive or as repulsive) to create the following potential energy diagram?

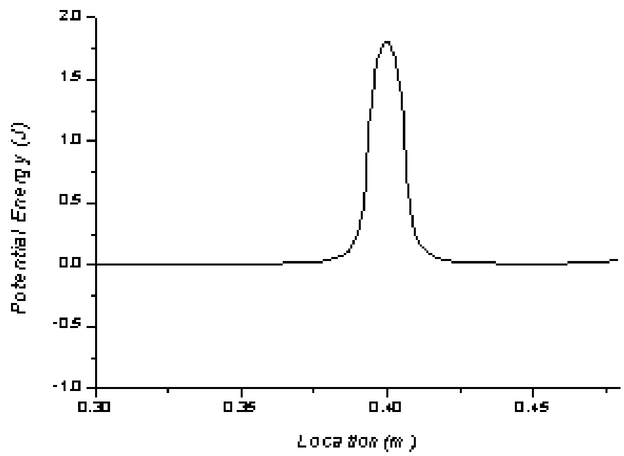


Figure 7: A potential energy diagram.

In general, when we work on a certain task, we always prefer to have as much information (data) available as possible. In the example with the car, to completely analyze its behavior, you will need an additional piece of information --- the total energy of the car. The total energy (*potential + kinetic*) is often represented by a horizontal line added either to the kinetic or potential energy diagrams. From now on we will draw the total energy as a line relative to the potential energy diagram, as shown in Figure 8.

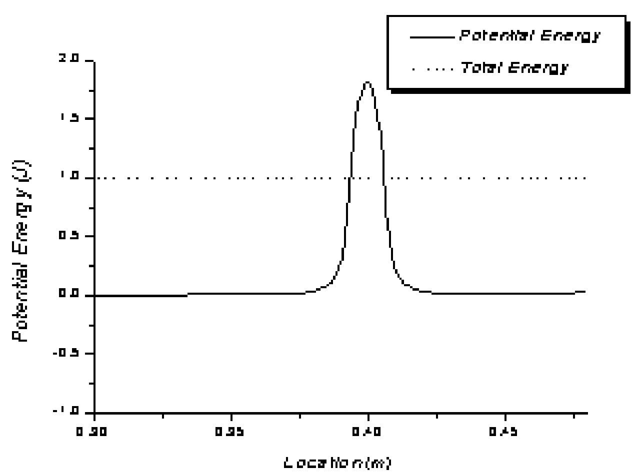


Figure 8: A potential and total energy diagram.

E-2. Why should we draw the total energy as a flat line, while the potential energy changes in situations where we ignore friction?

Consider a car which has the total energy indicated in Figure 9. It is approaching a set of magnets from the right that have the potential energy represented by the solid curve. See Figure 10.

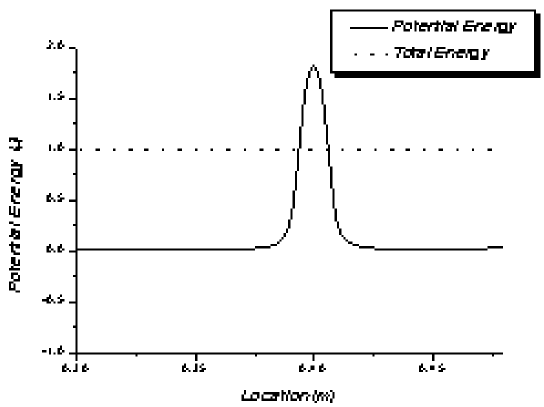


Figure 9: A potential and total energy diagram.

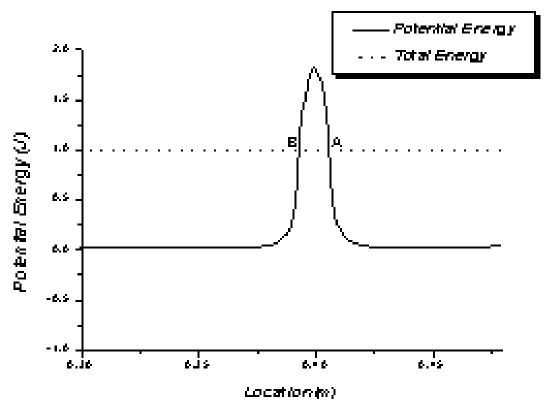


Figure 10: The magnets repel and the total energy is less than the maximum potential energy.

E-3. What is the value of the kinetic energy:

- at .44 m

- at point A, where the potential energy and total energy are equal

- at .40 m

Notice that in the region between points A and B the kinetic energy *that we calculate* is negative. This result is a difficulty. The mass of the car is always positive and so is the square of its velocity. Thus, kinetic energy can not be negative. A negative kinetic energy has no physical meaning! For a negative kinetic energy to exist, it has to be associated with negative mass or negative (velocity)^2. Neither of these is physically possible. The car cannot have a negative kinetic energy, so it does not go into regions where we would calculate a negative kinetic energy. Therefore, it is not physically possible for the car to get to the *left* of point A. When the car approaches point A from the right, it will stop at point A and turn back in the opposite direction.

Arrange the magnets so that you have a situation represented by the potential energy diagram in Figure 11.

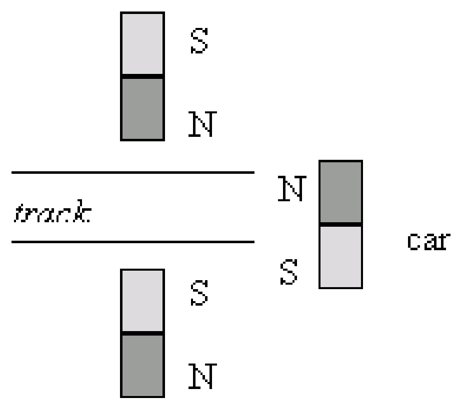
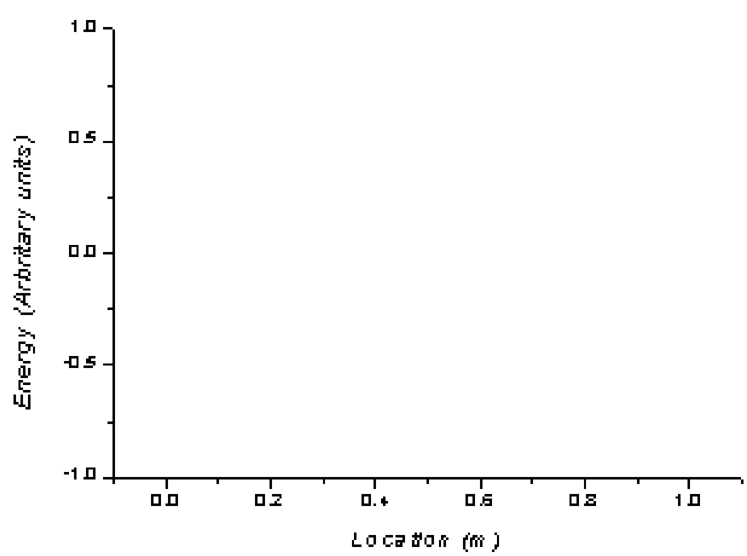
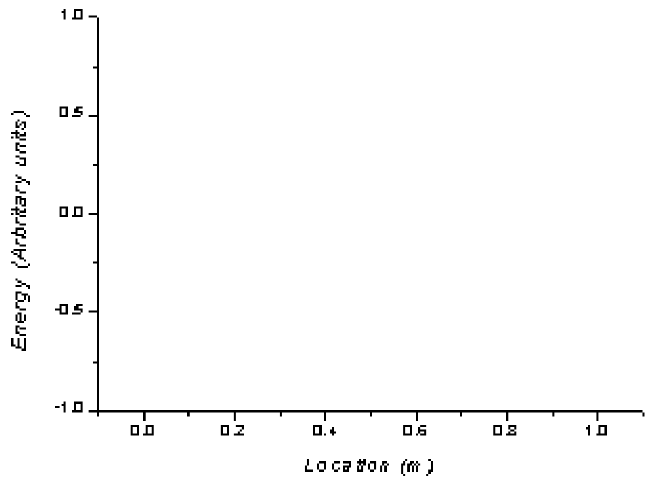


Figure 11: A situation in which the car is repelled by a pair of magnets.

E-4. Push the car, so that it turns around near the magnets. Then, sketch below the approximate values of the total energy and the potential energy. Mark the location at which the car turns around at point C.



E-5. Now give the car a stronger push. What does the car do when it approaches point C (i.e. near the magnet)? Sketch below both the total energy level of this car and its potential energy. Indicate on this graph the approximate location of point C from the previous graph.



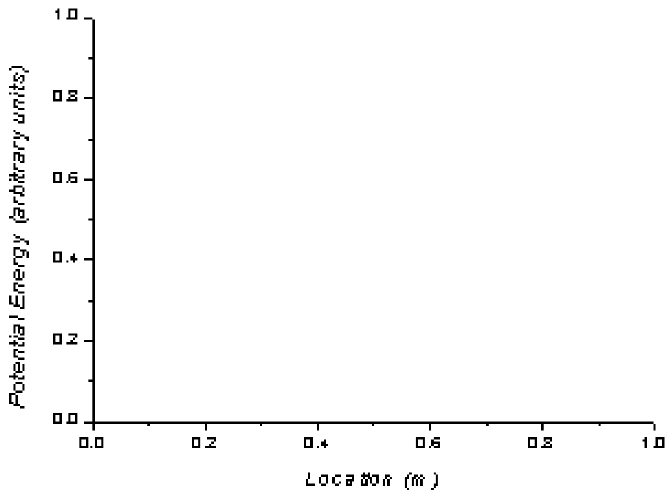
Notice that for some locations on the diagram the total energy of the car equals its potential energy. On one side the potential energy is greater than the total energy. This situation is equivalent to the kinetic energy having a negative value. Thus, the car will stay in the region where potential energy is equal to or less than the total energy. At the locations where the potential energy and total energy are equal, the car comes to a momentary stop, then changes direction. The locations where the car momentarily stops are called *turning points*.

F. Warning! The Car Is Approaching a Trap!

When we study atoms, we will look at several situations where two objects attract each other and, thus, stay close together. For example, an electron and a proton become a hydrogen atom because of their attraction. One way to learn about atoms is to consider the potential energy created by the attraction, then analyze the electron's total energy. To become prepared for that analysis we will now look at cars trapped by magnets.

F-1. We wish to establish a situation where the car is moving but it is trapped in a certain region of space. Arrange magnets (as many as you need) to create a situation where the car can be trapped in a region of space. Describe your arrangement of magnets.

F-2. Draw the potential energy diagram for this situation.



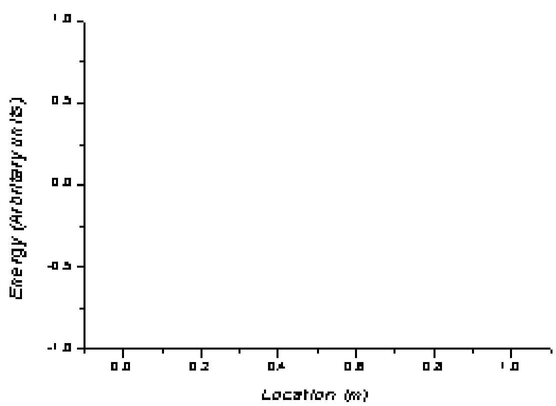
F-3. Use the potential energy diagram to explain why your arrangement will result in a trapped car.

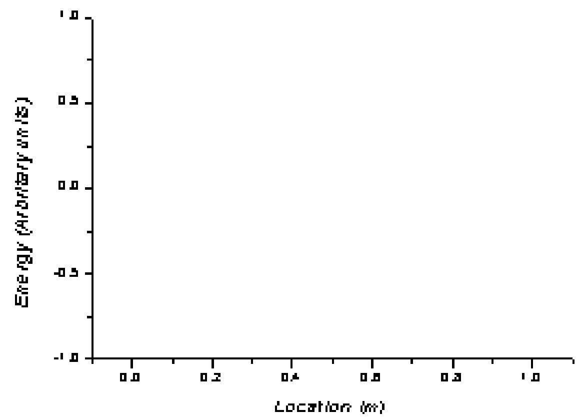
Test your arrangement to be sure it works. If you have any surprises, discuss them with your instructor.

F-4. Now give the car a push that allows it to escape from the trap that you have created. Use energy to explain why the car can escape.

F-5. Sketch your potential energy diagram again below. Then draw and label a possible total energy of the car when:

- it is trapped in one region of space and
- it escapes from the region.





F-6. The potential energy diagram in Figure 12 shows one possibility for getting a trapped car. On this diagram draw the maximum total energy level which the car can have and still remain trapped by two pairs of magnets. Mark the starting location of the car. Explain your answer.

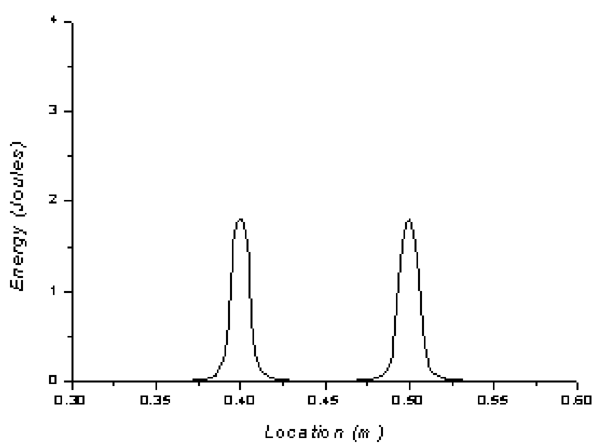


Figure 12: A potential energy diagram for a trapped car.

F-7. The diagram in Figure 13 shows potential and total energies for a car. Indicate on the graph the car's turning points.

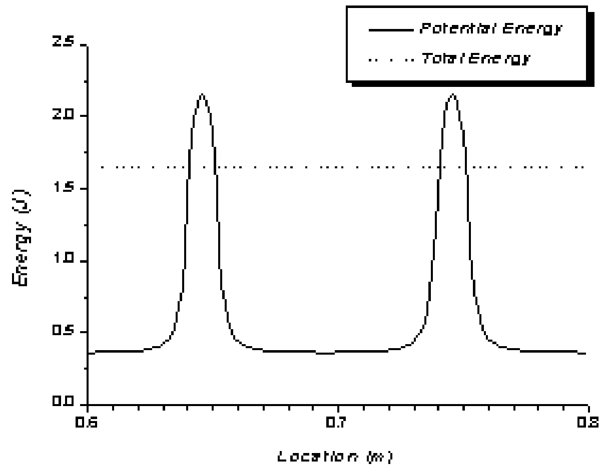


Figure 13: A potential energy diagram for a trapped car with total energy indicated.

Identifying the turning points is a useful way of describing the region in which an object is restricted. Using potential and kinetic energy is an easy way to determine the turning points.

G. Once Trapped, Will the Car Stay There Forever?

When an object is trapped in situations similar to the one represented by Figure 13, it must receive some energy if it is to get out of the trap. Suppose you wanted to help get this car out of its trap.

G-1. How much energy would you need to give the car with the total energy represented by Figure 13 so that it would no longer be trapped?

The energy needed to remove an object from a region in which it is trapped is called the *binding energy*. The binding energy is the *difference* between the maximum potential energy and the object's total energy.

H. Application — If Friction Were Not Present

Once again friction causes difficulty in seeing what happens when no energy leaves the car. To have a friction-free experience use the *Potential Energy Diagram Sketcher* program. Set up a situation that creates the potential energy diagram in Figure 13. Give the car the approximate total energy shown in Figure 13.

H-1. Describe the car's motion with

- large friction

- small friction

- no friction

H-2. Now, calculate the binding energy.

H-3. Give the car that much energy. What happens?

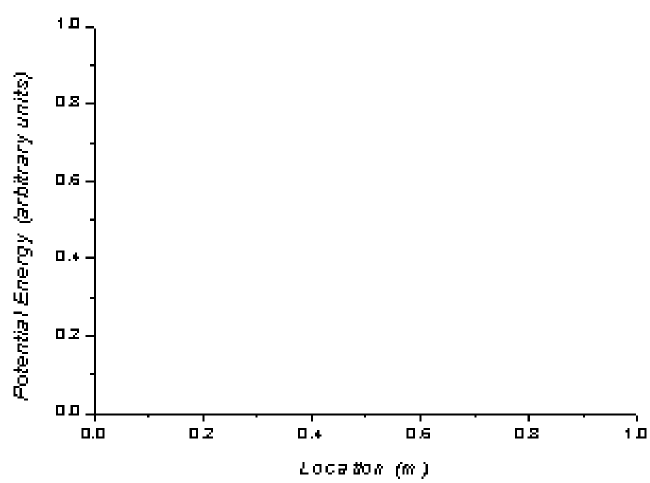
Each member of the group should set the total energy of the car to a different value. Other group members should calculate the binding energy. Then, see if that amount of energy frees the car from its trap.

The attractive and repulsive diagrams that you have created were diagrams of simple shapes - representing an object being first accelerated and later decelerated (or vice versa). However, the motion of an object is often more complex.The next section is an introduction to potential energy diagrams of complicated shapes and their application.

H-4. In the previous section we trapped a car by placing it between two locations where repulsive interactions occur. Now set up a situation so that the car is trapped by an attractive interaction. Describe below how you did it.

H-5. You also used an attractive interaction at the beginning of this tutorial. How is this situation different?

H-6. Remember the potential energy far from the magnets will be zero, and we define attractive potential energies to be negative. Sketch the potential energy diagram for the interaction that you have created. Again, assume that you can ignore friction. Draw only the potential energy due to the magnets.



H-7. Now, draw the total energy on this potential energy diagram as if friction were zero. To think about the value of the total energy, consider the turning points.

Based on our previous activities we must conclude that the total energy is negative when a single interaction traps an object. In this situation we have a negative potential energy and a negative total energy; only the kinetic energy is positive.

For many situations such as electrons in atoms, the object is trapped by one interaction. When objects are trapped in this way, we say they are in a *bound state*.

The binding energy for an object in a bound state can be determined as we did previously. It is the difference between the energy when the particle is *not* trapped and the energy when it is in the bound state.

H-8. Indicate the binding energy for the car on the graph in H-6.

This type of energy diagram is most useful when studying atoms. It will help us understand how energy changes in atoms and how light is created by atoms and what energy is needed to remove an electron from an atom.

I. Summary

In these activities we have presented energy as a method of describing motion. To use this method you need to create a potential energy diagram from the physical situation, then use it and knowledge of the total energy to determine other variables of the motion such as speed and acceleration. Conservation of energy is crucial to understanding the energy diagrams. When the total energy is constant, a decrease in potential energy means an increase in kinetic energy and vice versa. Thus, by looking at a potential energy diagram with the total energy marked on it, one can quickly use conservation of energy to describe the motion.

Inspection of energy diagrams quickly tells where an object such as a toy car can be. If a region exists where the potential energy is greater than the total energy, the object cannot enter that region. Because the object cannot be in a region, it must turn around. The *turning point* is the location at which the object turns. At this point the potential energy is equal to the total energy. If two turning points exist, the object is trapped in the region between them. By looking at an energy diagram you can easily determine the region in which the object is bound to a region of space.

When an object is bound, it can become free only if it receives additional energy. We can determine this energy by calculating the difference between the total energy and the maximum value of the potential energy. This difference is the *binding energy*.

The energy diagram method enables us to quickly determine several features of an object's motion. It will be very helpful when we study electrons — and we will find that electrons do not behave quite the same way as toy cars.