# Communication About Integrals as a Tool for Instruction and Assessment.

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Joshua Von Korff, Dehui Hu, and N. Sanjay Rebello
Kansas State University Physics Department

## 1. Research Questions

- RQ1: How can we assess students' understanding of integration in a physics context?
- RQ2: After collaborative learning, how will students talk about integration in a physics context?

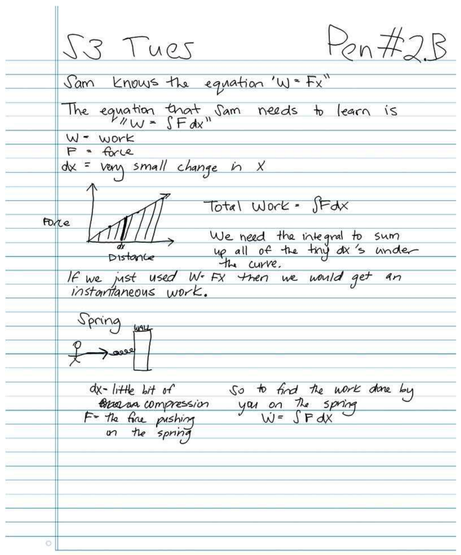
## 2. Methodology

- Participants: 16 students in an introductory calculus-based mechanics course.
- Six lessons, 1.5 hours each, over the course of 8 weeks.
- Students did not receive course credit, but were compensated financially.
- Students worked in groups of four. Lessons involved debate problems, group discussions using whiteboards, and presentations to other groups. (See box 4) Students gave one "Smartpen" presentation per lesson, which records both spoken and written information. (See box 5)

Lesson plan:

1. Kinematics: introduction to infinitesimals
2. Kinematics: the integral as a sum
3. Force and work: the integral as a sum
4. Mass and density: basics
5. Mass and density: a moment of inertia integral
6. Review, center of mass, and torque

## 5. Sample Student Work From Lesson 3




The student is told that "Sam" understands the equation "W = F x"; how can you explain to him why W = ∫ F dx? Excerpts below are from a student's presentation ("Kelly"):

Kelly: "The total work [is] the integral of force dx. So a little bit right here [shades in one rectangle], this is our dx. So we need the integral to sum up all of the tiny dx's under the curve."

Kelly says that she is integrating "force dx." However, she views the rectangle as "dx" rather than "dW", and says that "dx" must be summed, rather than "F dx."

Kelly: "In our picture we have dx is the little bit of force -- uh, the little bit of compression, and force is … the force pushing the spring."

"dx" is correctly described as "a little bit of compression." Kelly first writes "force," then scribbles this out.

## 3. Sample Debate Problem From Lesson 2

*(Starts with a story about a falling coffee filter, which students have seen in lab.)* A few students are talking about finding an equation for the *total* displacement of the coffee filter as it falls. Which ones, if any, are right? Which ones, if any, are wrong? Or are they partly right and partly wrong? How would you convince people when they are wrong?

a. Charles: "The equation is: $\Delta h \approx v_{total}\, t_{total}$ .
This is a quick way of summarizing the whole series of numbers."

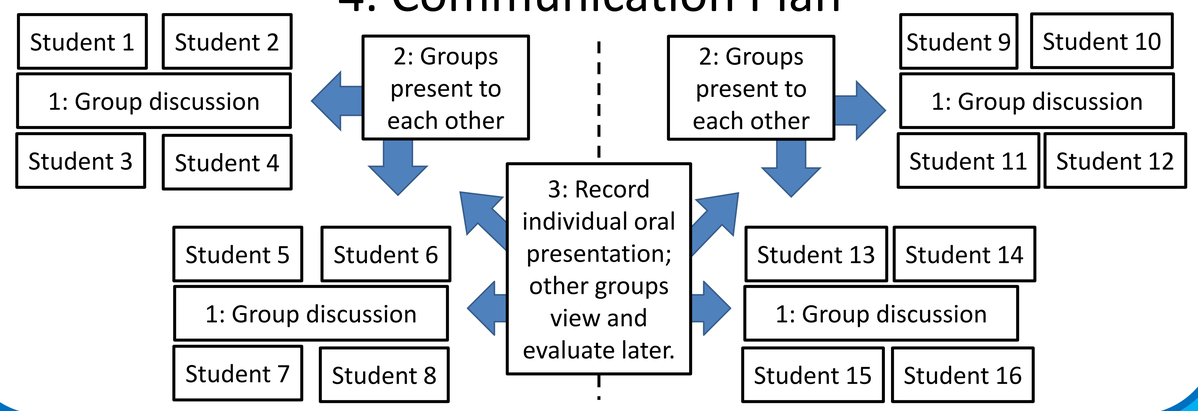
b. Sandy: "The equation is: $\Delta h \approx h_6 + v_1\, dt_1 + v_2\, dt_2 + v_3\, dt_3 + v_4\, dt_4 + v_5\, dt_5$ .
We are adding up the little displacements from the different small amounts of time."

c. Matt: "The equation is: $\Delta h \approx h_1 - v_1\, dt_1 - v_2\, dt_2 - v_3\, dt_3 - v_4\, dt_4 - v_5\, dt_5$ .
The velocities are negative because they point down."

d. Jackie: "The equation is: $\Delta h \approx t_1\, dv + t_2\, dv_2 + t_3\, dv_3 + t_4\, dv_4 + t_5\, dv_5$ .
This is basically the same as Sandy's equation, except it emphasizes the changing velocity."

e. Dawn: "The equation is: $\Delta h \approx \Sigma v_i\, dt_i$ .
This is just a shorthand way of writing the sum."

## 4. Communication Plan




## 6. Conclusions

- "Smartpen" presentations, which record both speech and writing, are a valuable tool for assessing students' understanding of integration in a physics context. Our previous research suggests that this kind of understanding is difficult to assess using quantitative problems.
- Using this tool, we can investigate how our students talk about integration. For instance:
  - Even after three lessons, many of our students have difficulty saying what physical quantity is being added up in an integral. When small amounts of work are being added up, students may incorrectly say that they are adding "force", "dx" (see box 5), or even "the force of the change in distance." Other students simply say they are summing "boxes" or "lines."
  - When learning about static spatial integrals for the first time, many of our students indicate that infinitesimals must refer to motions or "changes." According to their point of view, "dr" should not appear in an integral unless the physical situation involves a changing radius, r.