Students' Understanding of Differentials in Physics Integration Problems

Dehui Hu and N. Sanjay Rebello

Department of Physics, Kansas State University

This work is supported in part by the US National Science Foundation under grant 0816207.

1. MOTIVATION

- Students encounter several difficulties with setting up integrals in physics problems, especially interpreting differentials (or infinitesimals).[1,2,3,4]
- Our aim is to understand students' difficulties from the resources perspective. [5]

2. RESEARCH QUESTION

What mathematics and physics resources do students activate with the use of differentials in a physics context?

3. METHODOLOGY

- Group teaching/learning interviews
- 13 students from second-semester calculus-based introductory physics
- Students worked in groups of 2 or 3 with whiteboards
- Eight interview sessions that were 1 hour and 15 min each in length
- Interview tasks: physics integration problems in electricity & magnetism

4. RESOURCES ASSOCIATED WITH DIFFERENTIALS

Type of Resource	Example Quotes
"A very small amount" (i.e., an extremely small bit of a physical quantity)	"Well, we have a charge *Q* over the entire length of *L*, so this is just saying when you have *a little piece*, cause you can write it differently, you can write it as $\frac{dq}{dx}=\frac{Q}{L}$. So then it is just a ratio of a whole charge over the whole length to *a little bit of charge* over *a little bit of length*." (Refer to Figure 1 below)
"A point" (i.e., a point quantity or quantity of a point)	"The *charge at every single point* is charge divided by the distance." "Just find the little charges by taking the total charge over the length it's over, to find…since it's uniform, we can find the *charge at every point*." (Refer to Figure 1 below)
"Differentiation" (i.e., taking derivative of a function)	"So we need to *take the derivative* of it. So we can plug into *R*. Because we basically pull *dx* out of nowhere, because the *derivative of the only changing function*, then we require *dx.* We need to *integrate* that." "We have the *function*, you have to *take the derivative* so you can *take the integral.* I *don't know how to explain it other than mathematically."* (Refer to Figure 2 below)

1. An insulated thin rod with length L has charge +Q uniformly distributed over the rod. Point P is located at a distance d from the right end of rod. Find the electric field at point P due to this charged rod.

Fig. 1. Electric field due to a line of charge

2. A material with length L and cross-sectional area A lies along the x-axis between x=0 and x=L. Its resistivity varies along the rod according to $\rho(x)=\rho_0 \cdot e^{-x/L}$. Find the total resistance of this cylinder between two end faces.

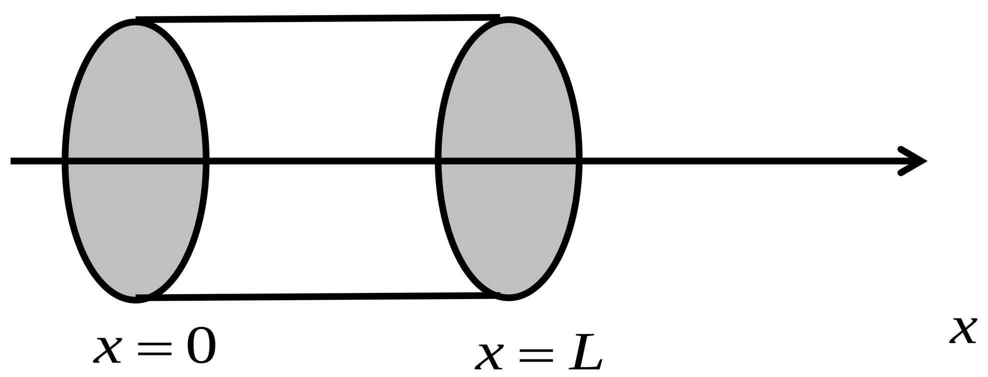

Fig. 2. Resistance of a cylindrical resistor with non-constant resistivity

5. CONCLUSION

- We identified three common resources students activate with the use of differentials:
  - "**A very small amount**" resource often refers to a small piece/segment of a physical quantity and is similar to what expert physicists often use.
  - "**A point**" resource is typically used when the physical size of an object can be neglected and we find that students have difficulty relating differentials to physical dimensions when this resource is activated.
  - "**Differentiation**" resource indicates that students consider "*d*" to be a mathematical operator without concrete physical meaning, leading them to invent an approach based on mathematical plug-and-chug.

REFERENCES

1. Yeatts, F.R., & Hundhausen, J.R., *Am. J. Physics,* 60 (8), 716, (1992).
2. Meredith, D.C., & Marrongelle, K.A., *Am. J. Physics*, 76 (6), 570 – 578 (2008).
3. Cui, L., Bennett, A.. Fletcher, P., & Rebello, N.S. Paper presented at the Annual Meeting of the National Association for Research in Science Teaching (2006).
4. Nguyen D. & Rebello N.S., Phys. Rev. ST Phys. Educ. Res. 7, 010112 (2011).
5. Hammer, D., American Journal of Physics - Physics Education Research Supplement, 68(7), S52-S59 (2000).