ON THE MOTION OF VORTEX PAIRS IN THE ANISOTROPIC HEISENBERG MODEL

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INTRODUCTION

We consider a classical 2D Heisenberg model with easy-plane symmetry. Kosterlitz and Thouless[1] showed that such a system has a topological phase transition: at low temperatures there exist bound vortex pairs which start to dissociate above a critical temperature T_{KT}. Just above T_{KT} we can assume that there are only a few free vortices which move ballistically between their interactions. A model of dynamics built on such a "vortex gas" has been constructed assuming a Gaussian velocity distribution.[2] Here we use effective equations of motion for the collective (center-of-mass) vortex variables and compare these analytical results of vortex-vortex and vortex-anitvortex interactions with molecular dynamics simulations of the full spin system.[3] We investigate both ferromagnets (FM) and antiferromagnets (AFM) with an anisotropy parameter λ varying from zero to one.

Theory

Our system is described by the Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z) , \tag{1}$$

with the sum running over all the nearest neighbor pairs in the plane and a positive exchange parameter J for the FM and a negative one for the AFM. Assuming Landau-Lifshitz dynamics we obtain two different single vortex solutions from the equations of motion:[2] for $\lambda < \lambda_c$ (FM: $\lambda_c \approx 0.72$; AFM: $\lambda_c \approx 0.71$) static vortices are purely in-plane; for $\lambda > \lambda_c$ an additional out-of-plane component develops, allowing a continuous crossover to the isotropic Heisenberg limit ($\lambda = 1$) where the topological excitations are merons and instantons.[4]

Following a general procedure for magnetic systems,[5] and allowing also some damping, an equation of motion of a single vortex in the presence of another vortex in the center-of-mass coordinates and the continuum limit is

$$\mathbf{G} \times \mathbf{v} + \mathbf{D}\mathbf{v} + \frac{\gamma}{m_0} \mathbf{F} = 0 . \tag{2}$$

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Here **G** and **D** are the gyrovector and the dissipation matrix, respectively, and both contain information about the actual vortex structure. **F** is the static force between the two vortices,[1] γ is the gyromagnetic ratio, and m_0 is the local magnetic moment per unit area.

Simulations

For the simulations we considered a 50×50 square lattice with free boundary conditions. The integration was performed with a fourth-order Runge-Kutta method with time step 0.04 $\hbar/JS$. For $\lambda < \lambda_c$ we initialized the simulations with two static in-plane vortices while for $\lambda > \lambda_c$ the four spins surrounding the vortex cores were given a small z component to guarantee the desired sign of the out-of-plane structure.

Ferromagnet

For $\lambda < \lambda_c$ the vortex is purely in-plane with small z components proportional to the velocity. For this case the gyrovector is vanishing and Eq. (2) predicts a motion of the vortices along straight lines: if the two vortices have equal vorticities they repel each other while a vortex and an antivortex attract each other. However, the simulations were performed on a discrete lattice which acts like a periodic "Peierls-Nabarro" pinning potential.[6] Thus the vortices have minimal energy if they are in the middle of a plaquette of four spins and they have maximal energy if they are at a lattice site. The resulting trajectories show therefore some fluctuations around the straight lines expected from Eq. (2). Moreover, the vortices will stop moving if their mutual distance is so large that the force between them is too weak to push them over the lattice potential. This scenario agrees very well with our simulations (Fig. 1).

For $\lambda > \lambda_c$ the vortices have a large ferromagnetic ordered out-of-plane structure which is extended over several lattice constants and which makes these vortices less sensitive to discreteness effects. The out-of-plane structure also acts like an effective magnetic field on the other vortex described by a nonzero gyrovector. In addition

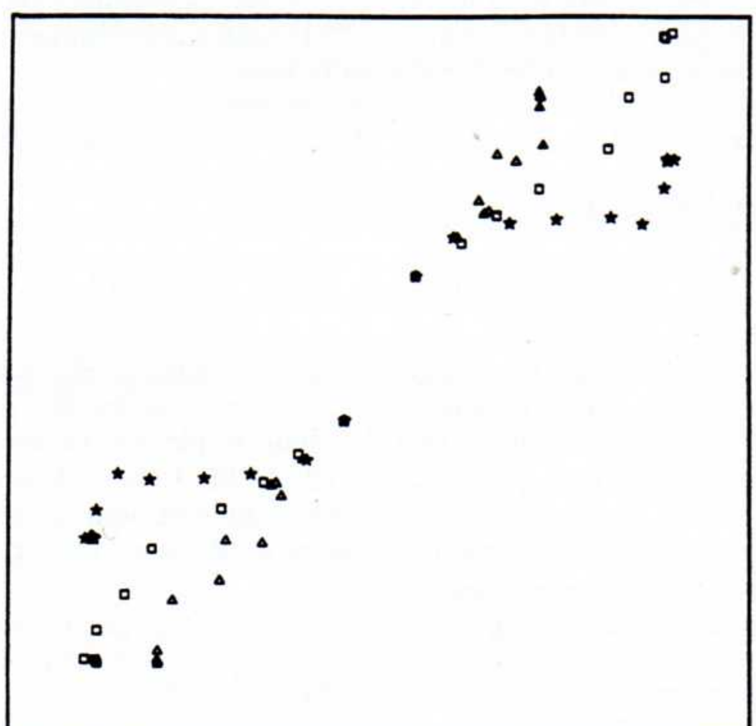


Fig. 1. Trajectories of vortex pairs on a FM with $\lambda < \lambda_c$ (square: $\lambda = 0.0$; triangle: $\lambda = 0.3$; star: $\lambda = 0.6$), $q_1 = q_2 = 1$ and initialized with two static in-plane vortices. Only a part of the lattice is shown: $18.0 \leq x, y \leq 30.0$, and the time between two successive points is four in our units.

each other (Fig. 2a) while for $p_1q_1 = -p_2q_2$ they move parallel to each other (Fig. 2b).

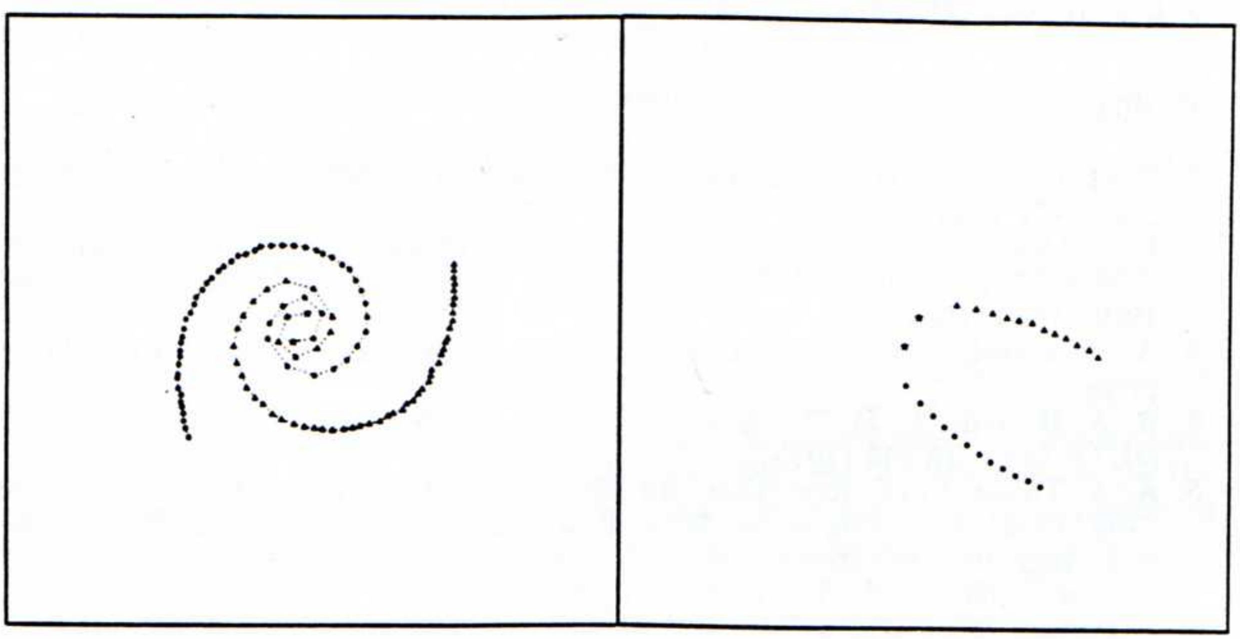
Antiferromagnet

In the AFM the static vortices have, for all λ, a local antiferromagnetic order, and deviations from this structure due to the vortex motion are in the same direction for adjacent spins.

For $\lambda < \lambda_c$ this gives almost the same behavior as in the FM for the same λ range, and the simulation yields a picture which is very similar to Fig. 1. For $\lambda > \lambda_c$, however, the behavior is quite different to the FM: here the out-of-plane structure is antiferromagnetically ordered which gives no contribution to the gyrovector. Thus, also for $\lambda > \lambda_c$ the vortices will feel only the static force between them and move on straight lines, but without the strong dependence on the discrete lattice (Fig. 3).

CONCLUSIONS

Considering pairs of unbound vortices in a classical easy-plane Heisenberg model we found that the resulting trajectories depend strongly on the anisotropy parameter λ and the exchange parameter J (FM or AFM). For $\lambda < \lambda_c$ the mainly in-plane structure gives a zero gyrovector in Eq. (2) and therefore the vortices move on straight lines caused by the static force (attraction or equal, repulsion for different vorticities). The discreteness effects are strong in our simulations which were performed at $T = 0$. For $T \gtrsim T_{KT}$, where we expect ballistically moving vortices, the thermal fluctuations should be large enough to cancel this lattice effect. For $\lambda > \lambda_c$



(a) (b)

Fig. 2. Trajectories of vortex-vortex simulations on a FM with $\lambda = 0.9, p_1 = p_2 = 1$ and initialized with two static out-of-plane vortices at positions (23.5, 23.5) and (24.5, 25.5) (star: start positions; circle: vortex 1; triangle: vortex 2); a) $q_1 = q_2 = 1$, the dashed line is a guide to the eye and connects successive points by straight lines; b) $q_1 = -q_2 = 1$.

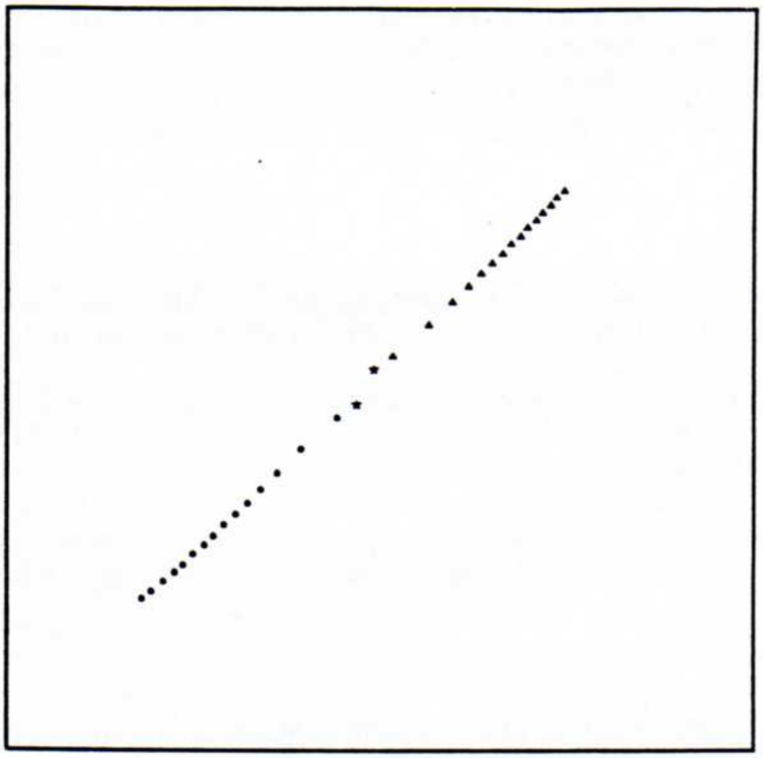


Fig. 3. Trajectories of a vortex-vortex ($q_1 = q_2 = 1$) simulations on an AFM with $\lambda = 0.9$ and initialized with two static in-plane vortices.

we must distinguish between $J > 0$ and $J < 0$. For the FM the out-of-plane structure results in an additional rotation to a translation of the vortices depending on the products $p_1 q_1$ and $p_2 q_2$. This behavior is similar to scenarios in 2D incompressible fluids,[7] superconductors[8] and superfluids.[9] In the AFM for $\lambda > \lambda_c$, however, the out-of-plane structure has no additional effect on the motion and we find only attraction or repulsion due to the static force.

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