Bayreuth workshop on Nonlinear Coherent Structures 1993

MASS AND MOMENTUM FOR VORTICES IN TWO-DIMENSIONAL EASY-PLANE MAGNETS

G. M. Wysin,[1] F. G. Mertens,[2] A. R. Völkel[2] and A. R. Bishop[3]

[1]Kansas State University, Manhattan, KS 66506 U.S.A.
[2]University of Bayreuth, D-8580 Bayreuth, Germany
[3]Los Alamos National Laboratory, Los Alamos, NM 87545 U.S.A.

INTRODUCTION

Many quasi-two-dimensional magnetic systems are expected to allow particle-like vortex excitations that are spin configurations with a net 2π twist about a particular point or core. Examples are XY or easy-plane quasi-2d magnets such as $BaCoAsO_4$, and the nearly Heisenberg-like K_2CuF_4 with a weak easy-plane anisotropy.[1] Vortices are also possible in systems such as monolayer magnetic lipid systems.[2] Vortices are created or destroyed in particle-antiparticle pairs, carry a conserved circulation charge, exert pair interaction forces on each other, and contribute to thermodynamics and spin-correlation functions.[3] There has been much interest in their role in a topological transition due to vortex-antivortex unbinding[4] above a characteristic temperature T_{KT}.

The vortex contributions to spin-correlations can be calculated approximately[5,6] by assuming a gas of weakly interacting vortices for temperatures above T_{KT}, with a Boltzmann velocity distribution characterized by an average thermal speed v_{rms}. The thermal speed was estimated by Huber from a velocity autocorrelation function based on a vortex equation of motion introduced by Thiele.[7] However, there are actually two different types of vortices possible in the easy-plane magnet, known as "in-plane" and "out-of-plane", depending on whether the out-of-easy-plane spin component is zero or nonzero for the stationary vortex.[8,9] The Thiele equation has been found to be inadequate to describe the in-plane vortices. A related point is that not all of the dynamic properties of magnetic vortices are fully understood, especially concerning the concept of vortex momentum.

For these reasons a new equation of motion for vortices has been proposed.[10] The principal physical effect included in the new equation is that a moving vortex can possess a mass. The spin profile for a moving vortex depends on the velocity. For example, the out-of-plane spin components increase with velocity, and this is responsible for the mass. A momentum can also be associated with the mass, and these carry consequences for the motion of interacting pairs of vortices.

Vortex dynamics is also determined by a second type of charge known as the gyrovector. The total gyrovector of the system is conserved for a continuum limit, but we find that it is not conserved when generalized to a discrete lattice, unlike the circulation charge. As a result vortex-antivortex annihilation can occur in a lattice system when it would be prohibited in the continuum system.

The new equation of motion with mass was discussed previously,[10] starting from a definition of momentum. Here we take an alternative approach where momentum need not be defined in order to obtain the dynamic equation, starting from the Landau-Lifshitz equation for the spin dynamics.[11] This will be followed by some discussion of the vortex momentum and problems with its definition. These results will be related to predicting motions of interacting pairs of vortices, as well as how to determine vortex masses from simulations. Simulations show that the gyrovector is not a conserved quantity for lattice systems. We begin by summarizing some of the properties of in-plane and out-of-plane vortices in the easy-plane ferromagnet.

2D EASY-PLANE FERROMAGNET AND VORTICES

We consider a Heisenberg model with ferromagnetic exchange $J > 0$ and easy-plane anisotropy characterized by $0 < \delta \leq 1$, with Hamiltonian

$$H = -J \sum_{(\mathbf{n},\mathbf{m})} (\vec{S}_\mathbf{n} \cdot \vec{S}_\mathbf{m} - \delta S^z_\mathbf{n} S^z_\mathbf{m}). \tag{1}$$

$\vec{S}_\mathbf{n}$ is a classical 3d spin vector at site $\mathbf{n}$ in a 2D square lattice, and the sum is over near-neighbor bonds. The limits $\delta = 0$ and $\delta = 1$ correspond to the isotropic Heisenberg and XY models, respectively. The individual spin length S is conserved, and the dynamic variables are the in-plane angle $\phi_\mathbf{n} = \tan^{-1}(S^y_\mathbf{n}/S^x_\mathbf{n})$ and its canonically conjugate momentum $S^z_\mathbf{n}$.

In a continuum limit, the lattice point $\mathbf{n}$ goes over into position $\vec{x}$, and $\vec{S}(\vec{x}, t)$ has dynamics given from Poisson brackets,

$$\frac{d\vec{S}}{dt} = \{\vec{S}, H\} = \vec{S} \times \vec{h}, \tag{2a}$$

$$\vec{h} = -\frac{\delta H}{\delta \vec{S}} = \nabla^2 \vec{S} - \delta(4S^z + \nabla^2 S^z)\hat{e}_z. \tag{2b}$$

Stationary Vortices

For static configurations, the in-plane angle ϕ satisfies Laplace's equation: $\vec{\nabla}\cdot\vec{\nabla}\phi = 0$. A vortex centered at position $\vec{X} = (X_1, X_2)$ causes a gradient

$$\vec{\nabla}\phi(\vec{x}) = \frac{q\hat{e}_z \times (\vec{x} - \vec{X})}{|\vec{x} - \vec{X}|^2}. \tag{3}$$

The circulation charge is $q = \pm$integer, for vortices (+) or antivortices (−). While no source term appears on the RHS of the Laplace equation for ϕ, we have

$$\vec{\nabla} \times \vec{\nabla}\phi = 2\pi q \delta(\vec{x} - \vec{X})\hat{e}_z. \tag{4}$$

This can be expressed in the form of a Gauss Law by rotating through 90^o, to make what is called the stream potential Ψ in fluid mechanics,

$$\vec{\nabla}\Psi = \vec{\nabla}\phi \times \hat{e}_z, \tag{5a}$$

$$\vec{\nabla}\cdot\vec{\nabla}\Psi = 2\pi q \delta(\vec{x} - \vec{X}). \tag{5b}$$

From these it is clear that q is a conserved charge with only integer values.

The out-of-plane component S^z determines the type of vortex. When the equations (2) are <u>linearized</u> in S^z, one finds $S^z \approx \dot{\phi}/J$, which gives $S^z = 0$ in the static limit and defines the static <u>in-plane</u> vortex. If the nonlinear terms in S^z are kept, the static out-of-plane vortex with $S^z \neq 0$ results. Far from the vortex core, the out-of-plane vortex has asymptotic form[9]

$$S^z \to p\sqrt{\frac{r_v}{|\vec{x} - \vec{X}|}}\, e^{-|\vec{x} - \vec{X}|/r_v}, \tag{6a}$$

$$r_v \equiv \frac{1}{2}\sqrt{\frac{1-\delta}{\delta}}. \tag{6b}$$

A natural length scale is determined by the vortex radius r_v, and $p = \pm 1$ determines the sign of S^z at the vortex core. Simulations[12] and energy estimates[9] have shown that only the in-plane vortex is stable for strong anisotropy ($\delta > 0.28$ for square lattice), while only the out-of-plane vortex is stable for weak anisotropy ($\delta < 0.28$ for square lattice).

Moving Vortices

When either type of vortex moves, perhaps due to the effect of other vortices, the out-of-plane spin component aquires a change S_1^z proportional to the vortex velocity $\vec{V}$ for low speed. Far from the vortex, with vortex position $\vec{X}(t) = \vec{V}t$, we have[9]

$$S_1^z \approx \frac{q\vec{V} \times (\vec{x} - \vec{X}(t))}{4J\delta|\vec{x} - \vec{X}(t)|^2} \cdot \hat{e}_z. \tag{7}$$

It is these structural changes in the vortex form that account for the generation of the mass to be discussed below.