Name______________________________________ KSU 2006/02/21

Instructions: Use SI units. No derivations here, just state your responses clearly, and define your variables in words.

1. (12) What are the general solutions for the vector and scalar potentials $\vec{A}(\vec{r},t)$ and $\Phi(\vec{r},t)$ when the sources $\vec{J}(\vec{r},t)$ and $\rho(\vec{r},t)$ have harmonic time dependence?

2. (12) In radiation problems, explain why you can usually obtain $\vec{E}$ and $\vec{H}$ by using only the vector potential $\vec{A}$. If you know $\vec{H}$, how do you get $\vec{E}$. Where does this really apply?

3. (12) A source of size d is emitting radiation at wavevector k. How do you define the

   a) near (static) zone; b) far (radiation) zone?

4. (12) The vector potential of an oscillating electric dipole is $\vec{A} = \frac{-i\omega\mu_0 e^{ikr}}{4\pi r}\vec{p}$.

   a) What is the definition of $\vec{p}$?

   b) Express the associated magnetic field $\vec{H}$ at arbitrary radius r.

5. (6) If the frequency of an oscillating electric dipole is doubled, by what factor will its total radiated power change?

6. (8) A radiation source produces certain fields $\vec{E}$ and $\vec{H}$ far from the source. How do you use them to obtain the power radiated per unit solid angle, along a direction $\hat{n}$?

7. (18) Jackson says the vector potential of an electric quadrupole is

$$\vec{A} = \frac{-\mu_0 c k^2}{8\pi} \frac{e^{ikr}}{r} (1 - \frac{1}{ikr}) \int d^3x' \; \vec{x}' \; (\hat{n} \cdot \vec{x}') \; \rho(\vec{x}').$$

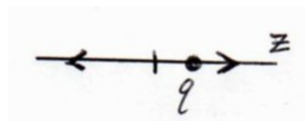
a) Write an expression for its magnetic field in the radiation zone.

b) How is its quadrupole tensor defined?

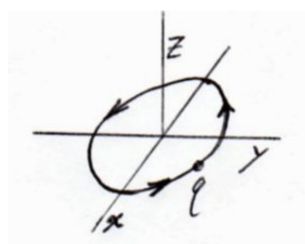
c) The total power from this expression is $P = \frac{c^2 Z_0 k^6}{1440\pi} \sum_{\alpha,\beta} |Q_{\alpha,\beta}|^2$. What approximations, if any, have been made to arrive at this formula?

8. (18) For the following sources, describe the predominant type of radiation (multipole and frequency), and the dependence of $dP/d\Omega$ on polar angle θ (between $\hat{n}$ and the z-axis).

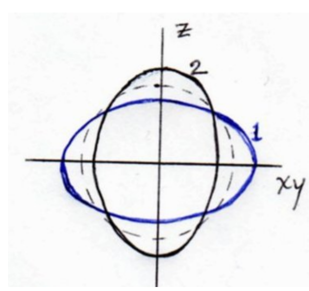
a) A point charge q moving sinusoidally at frequency ω_0 on the z-axis.



b) A point charge q rotating at angular velocity ω_0 in a circle of radius a in the xy-plane.



c) A spheroidal charge distribution with azimuthal symmetry around the z-axis, whose shape oscillates between the two extremes 1 and 2 as shown.



9. (12) Give a definition of differential scattering cross section, $d\sigma/d\Omega$. Be as complete as possible.

10. (12) What is the definition of a "vector spherical harmonic?" List two orthogonality properties that they have.

11. (8) When an expansion of a circularly polarized plane wave $\vec{E} = E_0(\hat{\epsilon}_1 \pm i\hat{\epsilon}_2)e^{ikz}$ is made, what possible multipoles (l, m) can be present?

12. (8) The total scattering cross-section of a small dielectric sphere is proportional to what powers of wavevector k and radius a?

13. (12) The differential scattering cross-section of a dielectric sphere is proportional to $|\hat{\epsilon}^* \cdot \hat{\epsilon}_0|^2$, where $\hat{\epsilon}_0$ and $\hat{\epsilon}$ are the incident and outgoing polarizations. For unpolarized incident light,

    a) At what scattering angle(s) θ (between incident and outgoing wavevectors) does $d\sigma_{\parallel}/d\Omega$ reach any extrema?

    b) At what scattering angle(s) θ (between incident and outgoing wavevectors) does $d\sigma_{\perp}/d\Omega$ reach any extrema?

14. (8) How do you define the "relative polarization" of scattered radiation, $\Pi(\theta)$? Be as specific as possible.

15. (18) Explain the symbols and application of the following formula:

$$\alpha = N\sigma_1 = \frac{2k^4}{3\pi N}|n-1|^2.$$

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Instructions: Use SI units. Please show the details of your derivations here. Explain your reasoning for full credit. Open-book and 1-page note summary allowed.

1. Linearly polarized plane wave radiation is incident on a free electron of charge $-e$, mass m. The amplitude is small enough so that the motion is nonrelativistic, and determined primarily by the electric field,

$$\vec{E}_{\rm inc} = E_0 \hat{\epsilon}_0 e^{i(k\hat{n}_0\cdot\vec{r}-\omega t)}.$$

   a) (12) Determine the motion of the electron from Newton's 2nd Law (use the harmonic approach, i.e., global $e^{-i\omega t}$ dependences).

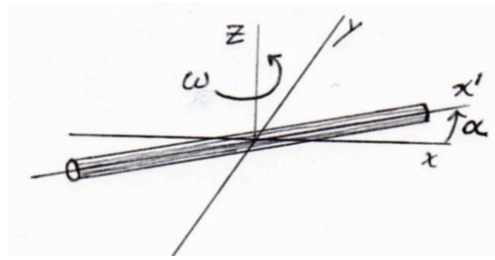
   b) (8) Find the time-dependent induced electric dipole moment.

   c) (24) Determine the scattered electric field $\vec{E}_{\rm sc}$ in the radiation zone.

   d) (24) Averaging over incident polarization $\hat{\epsilon}$, find the differential scattering cross sections $d\sigma_{\parallel}/d\Omega$ and $d\sigma_{\perp}/d\Omega$ for scattering within and perpendicular to the scattering plane.

   e) (16) Evaluate the total scattering cross section σ. What value (in meters) of classical electron radius r_c does it imply ($\sigma = \pi r_c^2$)?

2. A cylinder of length d and circular cross-section, radius a, contains a uniform volume charge density ρ. Here we consider the radiation it produces when rotated at angular velocity ω around an axis (z) through its center, perpendicular to the cylinder axis.



a) (20) [Optional!] Show that its electric quadrupole tensor in a $x'y'z'$ coordinate system fixed on the cylinder, with x' along the cylinder axis, and y', z', transverse, is

$$Q' = Q_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \qquad Q_0 = \rho\pi a^2 d \left(\frac{d^2}{6} - a^2 \right).$$

b) (20) [Also Optional!] The cylinder rotates through angle $\alpha = \omega t$ around the z axis as time progresses. Use the transformation properties for tensors of 2nd rank to show that the time-dependent quadrupole tensor (in lab frame) is

$$Q(t) = Q_0 \begin{pmatrix} \frac{1}{4}(1+3\cos 2\omega t) & \frac{3}{4}\sin 2\omega t & 0 \\ \frac{3}{4}\sin 2\omega t & \frac{1}{4}(1-3\cos 2\omega t) & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}.$$

c) (20) [Start Here] Use this $Q(t)$ to determine the complex quadrupole tensor to use in the analysis of the radiation. What will be the frequency of the emitted radiation?

d) (20) Find the radiated magnetic field $\vec{H}$ in the radiation zone. Give the xyz components of $\vec{H}$ as functions of the angular direction θ, ϕ of unit wave vector $\hat{n}$.

e) (20) Determine the angular distribution of radiated power, as a function of θ, ϕ.

f) (10) At a point on the x-axis in the radiation zone, along what direction is the radiation polarized?