Tutorial 3: Trapezoid rule

Brian Washburn, Version 1.0, 01/12/06

```
In[42]:= Off[General::spell];
```

The trapezoid rule is a very simple but powerful numerical method for integrating any function. Basically it breaks up the area under a function in small trapezoid shapes and add the areas of all these trapezoids. Let us integrate the following function from a=0, to b=10.

```
In[43]:= f[x_] = 300 x / (1 + Exp[x]);
         a = 0.0;
         b = 10.0;

In[46]:= exactsol = NIntegrate[f[x], {x, a, b}]

Out[46]= 246.59
```

The trapezoid rule looks like

$\int_a^b f(x)\,dx = (b-a) * \frac{f(a)+f(b)}{2}$

For n segments

$\int_a^b f(x)\,dx = \frac{h}{2}\,(f[a] + 2\sum_{k=1}^{n-1} f[a+k*h] + f[b])$ where h=(b-a)/n

We need to do the trapezoid rule with a large number of trapezoids (n segments). To verify this let us use the trapezoid rule for 2 and the 100 segments. The number of segments depends on the final error you want.

■ 2 segments

Lets do 2 segments, we can use this code for a general n.

```
In[47]:= n = 2;
         h = (b - a) / n;
         sol = h/2 (f[a] + 2 Sum[f[a + k * h], {k, 1, n - 1}] + f[b])

Out[49]= 50.5369
```

Compare with the exact solutions, two segments give a very poor error.

In[50]:=
$$\frac{\text{exactsol} - \text{sol}}{\text{exactsol}} * 100$$

Out[50]= 79.5057

■ 100 segments

Lets do 100 segments, we can use this code for a general n.

In[51]:=
$$n = 100;$$
$$h = \frac{b - a}{n};$$
$$sol = \frac{h}{2}\left(f[a] + 2\sum_{k=1}^{n-1} f[a + k * h] + f[b]\right)$$

Out[53]= 246.465

Compare with the exact solutions, which is less than 0.1% error

In[54]:=
$$\frac{\text{exactsol} - \text{sol}}{\text{exactsol}} * 100$$

Out[54]= 0.0507328