Tuitorial 2: Differential equations in *Mathematica*: Numeric solutions

Brian Washburn, Version 1.0, 01/08/06

```
Off[General::spell];
```

■ Differential equations solved numerically in *Mathematica*

Let us solve the same differential equation we solved using an analytic method here numerically. To solve the DE numerically we cannot have any undefined constants. So define a and ω, and solve the DE x''[t]=$-\omega^2$ x[t] with the boundary conditions, x[0]=a and x'[0]=0. We will used the command NDSolve[].

```
a = 0.1;
ω = 2 π 10;
```

```
? NDSolve
```

```
NDSolve[eqns, y, {x, xmin, xmax}] finds a numerical solution to the ordinary
   differential equations eqns for the function y with the independent variable x in
   the range xmin to xmax. NDSolve[eqns, y, {x, xmin, xmax}, {t, tmin, tmax}] finds a
   numerical solution to the partial differential equations eqns. NDSolve[eqns, {y1,
   y2, ... }, {x, xmin, xmax}] finds numerical solutions for the functions yi. More…
```

```
solx = NDSolve[{x''[t] == -ω^2 x[t], x[0] == a, x'[0] == 0.0}, x, {t, -0.1 π, 0.1 π}]
```

```
{{x → InterpolatingFunction[{{-0.314159, 0.314159}}, <>]}}
```

Notice that we need a range of time values {t,-0.1 π,0.1 π}. This is because the output is not an analytic function! The output is really a list of x values for a given range of times (-0.1 π seconds to 0.1 π seconds). The Interpolation function is this "list". So, when you evaluate x[t] it looks up the value in the interpolation function for that value of t. Again, we have the function to something we can plot.

```
xsol[t_] := x[t] /. solx[[1]];
```

Let us check the numerical solution by testing the boundary conditions!

```
xsol[0]
```

```
0.1
```

Which is a! To find the velocity, we can take the derivative of xsol[t]. The result is a new interpolation function.

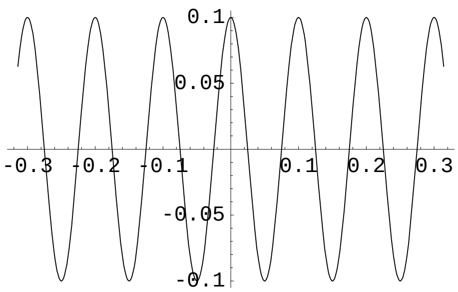
```
vxsol[t_] = ∂t xsol[t]
```

```
InterpolatingFunction[{{-0.314159, 0.314159}}, <>][t]
```

```
vxsol[0]
```

```
0.
```

```
Plot[xsol[t], {t, -0.1 π, 0.1 π}];
```



This plot is the same as the analytic result!

REMEMBER, WHEN DOING NUMERICAL SOLUTIONS ALWAYS FIND SOME WAY TO CHECK YOUR RESULTS