Introduction to *Mathematica*: Analytic and numerical integration

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This notebook is to help introduce you to *Mathematica* by computing an integral analytically and numerically. Let's do something simple to start.

```
In[5]:= 2 + 2
Out[5]= 4
```

To run the command "2+2" type shift-Return or hit the Enter key on the keypad.

■ Simple integral that can be done analytically

Now,let us define a simple function to integrate

```
In[8]:= f[x_] = Sin[x]
Out[8]= Sin[x]
```

Determine the integral analytically, using a three ways

```
In[10]:= ∫Sin[x] dx
Out[10]= -Cos[x]

In[11]:= ∫f[x] dx
Out[11]= -Cos[x]

In[21]:= Integrate[f[x], x]
Out[21]= -Cos[x]
```

The ? is used to find the help on commands. All commands are of the form Command[input1,input2,...].

```
In[22]:= ?Integrate

        Integrate[f, x] gives the indefinite integral of f with respect
           to x. Integrate[f, {x, xmin, xmax}] gives the definite integral of f with
           respect to x from xmin to xmax. Integrate[f, {x, xmin, xmax}, {y, ymin,
           ymax}] gives a multiple definite integral of f with respect to x and y. More...
```

OK, lets find a numerical value by using some limits a couple of ways

```
In[13]:= ∫_0^π f[x] dx
Out[13]= 2
```

```
In[23]:= Integrate[f[x], {x, 0, Pi}]
Out[23]= 2
```

To check the result, use the result above at the limits of zero and pi.

```
In[15]:= -(Cos[π] - Cos[0])
Out[15]= 2
```

Notice we can define the resulting function of the integral as another function h[x]

```
In[29]:= h[x_] = ∫f[x] 𝕕x;
          h[y]
          h[π/4]
Out[30]= -Cos[y]
Out[31]= -1/√2
```

■ Complex integral that cannot be done analytically

Let's do a more difficult integral

```
In[19]:= g[x_] = Sin[x]/√(1 + x^2);
```

Integrate this function analytically

```
In[20]:= ∫g[x] 𝕕x
Out[20]= ∫ Sin[x]/√(1 + x^2) 𝕕x
```

When *Mathematica* does this, that means it cannot do the integral analytically. This means we must do a numerical integral using limits. Use NIntegrate and evaluate the integral from 0 to π.

```
In[24]:= ?NIntegrate
        NIntegrate[f, {x, xmin, xmax}] gives a numerical
           approximation to the integral of f with respect to x from xmin to xmax. More...
In[25]:= NIntegrate[g[x], {x, 0, Pi}]
Out[25]= 1.14534
In[32]:= NIntegrate[Sin[x]/√(1 + x^2), {x, 0, Pi}]
Out[32]= 1.14534
```