

# Fragmentation of $H_2^+$ in strong 800-nm laser pulses: Initial-vibrational-state dependence

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(Received 17 December 2002; published 15 April 2003)

The fragmentation of the  $H_2^+$  molecular ion in 25-fs, 800-nm laser pulses in the intensity range 0.05–0.5 PW/cm<sup>2</sup> is investigated by means of wave-packet propagation calculations. We use a collinear reduced-dimensionality model that represents both the nuclear and electronic motion by one degree of freedom including non-Born-Oppenheimer couplings. In order to reproduce accurately the properties of the “real” three-dimensional molecule, we introduce a modified “soft-core” Coulomb potential with a softening function that depends on the internuclear distance. The analysis of the calculated flux of the outgoing wave packets allows us to obtain fragmentation probabilities and kinetic-energy spectra. Our results show that the relative probabilities for dissociation and Coulomb explosion depend critically on the initial vibrational state of the molecular ion.

DOI: 10.1103/PhysRevA.67.043405

PACS number(s): 33.80.Rv, 33.90.+h, 42.50.Hz

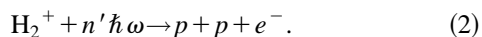
## I. INTRODUCTION

This simplest molecule, the hydrogen molecular ion, is of fundamental interest in atomic and molecular physics. It constitutes a bound three-body Coulomb system which, in contrast to its atomic counterparts, the negative hydrogen ion and the helium atom, exhibits (if we neglect rotation) two time scales, the fast electronic motion (as scale) and the vibrational motion of the nuclei [femtosecond (fs) scale]. The interaction of this smallest molecule with strong fs laser pulses is of particular interest, since the pulse duration is comparable to the vibrational period (14 fs for  $H_2^+$  in the vibrational ground state). Further, from a theoretical point of view, the simple structure of  $H_2^+$  allows for a numerical solution of the time-dependent Schrödinger equation within reduced dimensionality models by using, e.g., standard Crank-Nicholson split-operator propagation techniques [1–7].

Interacting with a strong, linearly polarized laser field,  $H_2^+$  exhibits two fragmentation channels, dissociation



and Coulomb explosion (CE)



The fragmentation of  $H_2^+$  in strong laser fields has been considered in numerous theoretical investigations emphasizing the influence of the two-center nature of the electronic potential on strong-field ionization [3,4,8–15]. Here, the laser-induced coupling of the lowest electronic states  $1s\sigma_g$  and  $2p\sigma_u$  results in a localization of the electron near one of the nuclei. The electron cloud oscillates between the nuclei and gives rise to dissociation of the molecule and to “charge resonance enhanced ionization” (CREI) at intermediate internuclear distances  $R$  between 5 and 10 a.u. [4,8,9].

Experimentally, the situation is more challenging since mostly neutral  $H_2$  targets are available [16–22], such that the production and fragmentation of the molecular ion occur in the same laser pulse and are difficult to separate. It was shown that for 50-fs pulses, saturation of  $H_2^+$  is never reached at any intensity [19]. Hence, there is a growing interest in molecular ions as targets which have been investigated in two recent experiments [23,24]. In this work, we consider the vibrational-state dependence of the fragmentation yields for the processes (1) and (2). The experimental data of Posthumus *et al.* [19] show that there is competition of dissociation and CREI for intensities around  $10^{14}$  W/cm<sup>2</sup>. We use wave-packet propagation calculations in a collinear  $2 \times 1D$  (one-dimensional) model to derive dissociation and CE probabilities and the corresponding kinetic-energy spectra for the fragments.

This paper is structured as follows. First, in Sec. II, we describe our theoretical model with emphasis on the effects of the reduced dimensionality on the potential curves of  $H_2^+$ . This section includes a discussion of the properties of the widely used “soft-core” (SC) Coulomb potential [25] for the electron-nucleus interaction (Sec. II A). In Sec. II B, we present an improved SC Coulomb potential which allows the exact reproduction of the 3D ground-state potential curve of  $H_2^+$ . Next, we provide kinetic-energy spectra by applying the recently proposed “virtual detector” method [26] to the laser-induced fragmentation of  $H_2^+$ . We present and discuss the numerical results for the vibrational-state dependence of the fragmentation dynamics including the intensity dependence for two selected vibrational states (Sec. III). Our concluding remarks follow in Sec. IV. Atomic units are used throughout this paper, unless otherwise indicated.

## II. THEORETICAL MODEL

For linearly polarized nonrelativistic laser fields considered here, only the coordinates parallel to the electric-field vector play an important role. Thus, in order to keep the computational effort at a reasonable level, we describe the molecular ion in a collinear model. This reduced-dimensionality model includes two coordinates, the elec-

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tronic coordinate  $z$  relative to the center of mass of the nuclei and the internuclear distance  $R$ . The model-inherent alignment of the molecule along the polarization axis is not a serious problem, since in a kinematically complete experiment fragmentation along this axis can be selected. We note that the model includes non-Born-Oppenheimer couplings between the electronic and nuclear motion. The solution of the time-dependent Schrödinger equation is formally given by the propagation of the initial wave function,

$$\Psi(z, R, t) = \exp\left[-i \int_0^t dt' \mathcal{H}(z, R, t')\right] \Psi(z, R, t=0) \quad (3)$$

with the Hamiltonian

$$\mathcal{H}(z, R, t) = \mathcal{T}_R + V_{nn}(R) + \mathcal{H}_{el}, \quad (4a)$$

$$\mathcal{H}_{el} = \mathcal{T}_z + V_{en}(z, R) + F(t)z. \quad (4b)$$

$\mathcal{T}_R$  and  $\mathcal{T}_z$  are the kinetic-energy operators for the electronic and the nuclear motion.  $V_{nn}(R) = 1/R$  is the internuclear Coulomb potential and  $V_{en}(z, R)$  the two-center Coulomb potential for the electron-nucleus interaction. The interaction of the electric component  $F(t)$  of the laser field with the electron is given in dipole approximation in the length gauge. The propagation (3) is carried out numerically on a grid using the Crank-Nicholson split-operator method which provides unconditional numerical stability [27]. Here, for each time step  $\Delta t$ , the wave function at time  $t + \Delta t$  is recursively given by

$$\begin{aligned} \Psi(t + \Delta t) &= \exp\left[-\frac{1}{2}i\mathcal{T}_R\Delta t\right] \exp\left[-i(V_{nn} + \mathcal{H}_{el})\Delta t\right] \\ &\times \exp\left[-\frac{1}{2}i\mathcal{T}_R\Delta t\right] \Psi(t) + O(\Delta t^3). \end{aligned} \quad (5)$$

In order to avoid the singularity of the Coulomb potential in 1D calculations, usually a SC Coulomb potential is introduced [3,25].

### A. Soft-core Coulomb potential for $\text{H}_2^+$

Before we discuss the application of the SC potential to the  $\text{H}_2^+$  molecular ion, we briefly consider its basic properties. The SC Coulomb potential is given by

$$V_{\text{SC}}(z) = \frac{-Q}{\sqrt{z^2 + a^2}}. \quad (6)$$

$Q$  describes the (effective) nuclear charge of the atom/ion, and  $a$  is a ‘‘softening’’ parameter which removes the singularity at  $z=0$ . For large  $z$ , this looks like a Coulomb potential  $-Q/z$  whereas the behavior for small  $z$  resembles a harmonic-oscillator potential

$$V_{\text{SC}}(z) = -Q/a + Qz^2/(2a^2) + O(z^3). \quad (7)$$

The parameter  $a$  can be adjusted in order to reproduce the correct ground-state energy. In principle, one could also change the parameter  $Q$ . However, this would lead to an unphysical asymptotic behavior of the potential, and  $Q$  would differ from the asymptotic charge seen by the electron

at large distances  $z$ . For one-electron systems, such as hydrogenlike ions, it turns out that for each value of  $Q$  the parameter  $a$  has to be adjusted separately in order to reproduce the correct binding energy of the ground state.

In the case of  $\text{H}_2^+$ , the two-center potential requires a continuous adjustment of  $a$  while the internuclear distance  $R$  is changed. The two limiting cases are  $R=0$  (He atom,  $a=0.709$ ) and  $R \rightarrow \infty$  (H atom,  $a=1.415$ ). In their previous work, Kulander, Mies, and Schafer [3] suggested a softening function  $a(R)$ , but did not use it for their calculations. Instead, they keep  $a=1$  fixed for all internuclear distances. They compare the basic properties of their 1D model molecule with known 3D results for the shape of the two lowest electronic state ( $1s\sigma_g$  and  $2p\sigma_u$ ) potential curves and the dipole matrix element  $d_{ug} = \langle 2p\sigma_u | z | 1s\sigma_g \rangle$  coupling these states. It has been shown [4,8,9] that these two states and their coupling are most important for the dynamics of  $\text{H}_2^+$  in strong laser fields. The degeneracy of  $1s\sigma_g$  and  $2p\sigma_u$  electronic states enables charge localization at one nucleus and results in a diverging dipole moment which increases linearly with  $R$ . The shape of the potential curves also affects the positions of (avoided) crossings of the different  $n\omega$  Floquet channels, corresponding to a net absorption of  $n=1, 2$ , or 3 photons [16]. Figure 1(a) shows these 1D dressed potential curves for a fixed softening parameter  $a=1$  in comparison with the result of a full 3D calculation. We find the  $1s\sigma_g$  curve in the 1D model from Ref. [3] shifted to larger  $R$  by 30% in the equilibrium distance whereas the  $2p\sigma_u$  curves show a better overall agreement of the 1D and 3D calculations. The larger equilibrium distance  $R_0$  in Kulander’s model is crucial, since the ionization probability depends critically on the position of  $R_0$  relative to the curve crossings. A comparison of the model molecular properties with the corresponding exact 3D values is given in Table I. The anharmonicity constant  $\omega_e x_e$  is obtained by fitting the lowest vibrational energies in the  $1s\sigma_g$  potential curve to the expression  $E_v = \omega_e(v + \frac{1}{2}) - \omega_e x_e(v + \frac{1}{2})^2$ . Figure 2 shows the 1D and 3D values for the coupling dipole moment  $d_{ug}$  of the lowest electronic states as a function of  $R$ .

### B. Improved soft-core Coulomb potential for $\text{H}_2^+$

Based on the idea of Kulander, Mies, and Schafer [3] to use a softening function  $a(R)$ , we now attempt to find a more realistic 1D model. The SC Coulomb potential (7) behaves like a harmonic-oscillator potential for small  $z$ . The parameter  $a$  determines simultaneously the depth  $-Q/a$  and width of the potential. The width is proportional to  $a/\sqrt{Q}$ , i.e., it is given by the prefactor  $Q/a^2$  of the quadratic term. Our idea is now to control depth and width of the SC Coulomb potential for  $\text{H}_2^+$  independently in order to reproduce the exact  $1s\sigma_g$  curve and, at the same time, to yield a reasonable agreement for  $2p\sigma_u$ . This modified SC Coulomb potential is given by

$$V_{\text{SCmod}}(z) = \frac{-Q}{1/a - a/b + \sqrt{z^2 + (a/b)^2}}, \quad (8)$$

with two parameters  $a$  and  $b$ . For small  $z$  we find

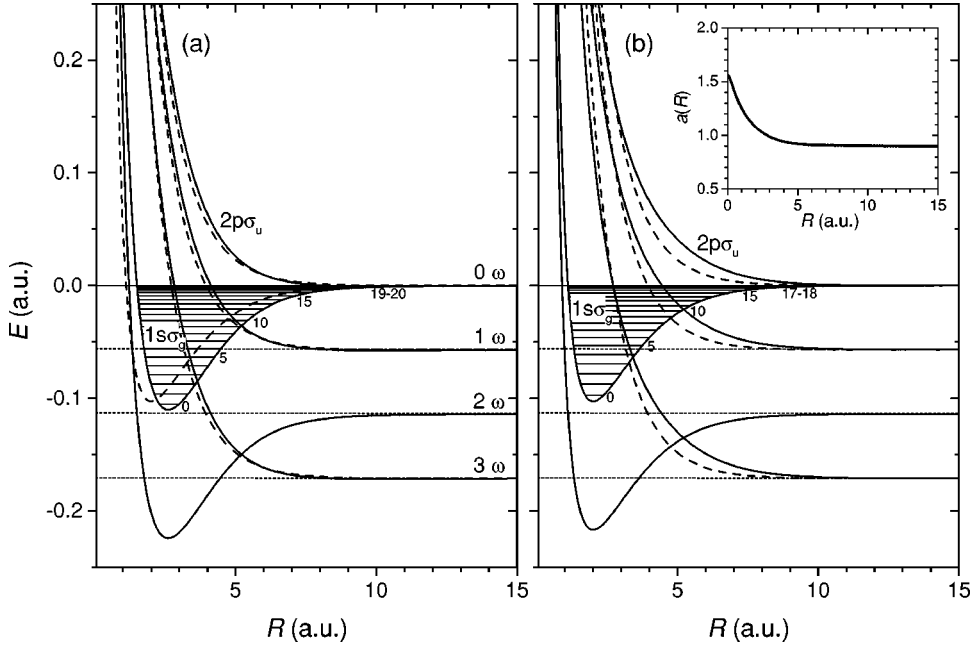


FIG. 1. Dressed  $n\omega$  ( $n = 0, \dots, 3$ ) Floquet potential curves and vibrational energy levels for the two lowest electronic states  $1s\sigma_g$  and  $2p\sigma_u$  of  $H_2^+$ . Solid curves, 1D model molecule; dashed curves, full 3D result. (a) Model by Kulander, Mies, and Schafer [3] using the SC Coulomb potential (6) with a fixed softening parameter  $a=1$ . (b) Improved model (this work) based on the modified SC Coulomb potential (8) with the softening function  $a(R)$  (shown in the inset).

$V_{SCmod}(z) \approx -Q(a - \frac{1}{2}bz^2)$ , whereas for large  $z$  the asymptotic behavior  $-Q/z$  is unchanged. Using  $V_{SCmod}$ , the two-center model potential in Eq. (4) for  $H_2^+$  ( $Q=1$ ) takes the form

$$V_{en}(z, R) = \frac{-1}{1/a(R) - a(R)/b + \sqrt{z_-^2 + [a(R)/b]^2}} + \frac{-1}{1/a(R) - a(R)/b + \sqrt{z_+^2 + [a(R)/b]^2}},$$

$$z_{\pm} = z \pm R/2. \quad (9)$$

For a given value of  $b$ , the function  $a(R)$  is adjusted to *exactly* reproduce the 3D potential curve of  $1s\sigma_g$ .  $b$  is a

fixed parameter which determines the shape of the repulsive  $2p\sigma_u$  curve. The result for  $b=5$  in comparison with the 3D curves is shown in Fig. 1(b) with the function  $a(R)$  given as an inset. There is still some difference between our 1D and the full 3D models for the  $2p\sigma_u$  potential curve. A better agreement can be achieved for larger values of  $b$  but this makes the potential more and more cusplike at the position of the nuclei. This limits the value of  $b$  since for practicable grid spacings the numerical grid cannot resolve a very narrow potential cusp. We found that  $b=5$  models the repulsive  $2p\sigma_u$  potential curve well without causing spurious effects in the numerical propagation on the grid. Based on our modified SC Coulomb potential (9), we calculated the properties of our 1D model molecule. In comparison with the 1D SC Coulomb potential from Ref. [3], our results are in better agreement with the 3D data (see Fig. 1 and Table I). In

TABLE I. Properties of the 1D model molecule compared with the real 3D molecule (atomic units).  $R_0$ , equilibrium internuclear distance;  $D_e$ , dissociation energy;  $\omega_e$ , ground-state frequency;  $\omega_e x_e$ , anharmonicity constant;  $I_p$ , ionization potential.

		1D model		
		3D result <sup>a</sup>	[3]	This work
$R_0$		2.0	2.6	2.0
$D_e$		0.103	0.11	0.103
$\omega_e$		$1.06 \times 10^{-2}$	$9.93 \times 10^{-3}$	$1.06 \times 10^{-2}$
$\omega_e x_e$		$3.02 \times 10^{-4}$	$2.02 \times 10^{-4}$	$3.02 \times 10^{-4}$
$I_p(R_0)$		1.1	1.154	1.1
Number of bound states		19	21	19
$1\omega$ crossing	$R$	4.8	5.2	5.2
	$\nu$	9	9	10
$3\omega$ crossing	$R$	3.3	3.6	3.4
	$\nu$	3	2	4
$1\omega$ threshold		$\nu \geq 5$	$\nu \geq 6$	$\nu \geq 5$

<sup>a</sup>Molecular constants from Ref. [28].

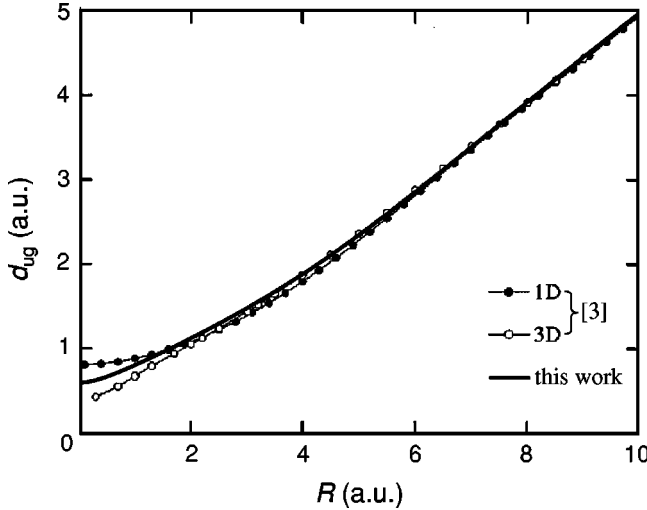


FIG. 2. Dipole coupling matrix element  $d_{ug} = \langle 2p\sigma_u | z | 1s\sigma_g \rangle$  for the lowest electronic states of  $\text{H}_2^+$ . One-dimensional model calculation of this work compared with the former result (1D model and full 3D calculation) from Ref. [3].

addition, the dipole moment  $d_{ug}$  is slightly better reproduced than by the simple SC Coulomb potential (Fig. 2).

We note that by adjusting a softening function  $a(R)$  only in a model potential  $V(z, R) = -Q/\sqrt{z^2 + a(R)^2}$ , the  $1s\sigma_g$  potential curve can also be exactly reproduced. However, the shape of the corresponding repulsive  $2p\sigma_u$  curve is in quite poor agreement with the 3D result. This led us to introduce a second free parameter  $b$  in Eqs. (8) and (9).

### C. Numerical propagation scheme

Our numerical grid extends from 0.05 to 30 in  $R$  with a spacing of 0.05 and from  $-45$  to  $45$  in  $z$  with a spacing of 0.2. Absorbing regions using an optical potential are introduced in order to prevent the reflection of the outgoing wave packets at the border of the grid. We implemented this absorber as fourth-order optical potentials in  $z$  and  $R$ :

$$V_{\text{opt}}(z, R) = -i[V_z(z) + V_R(R)], \quad (10a)$$

$$V_z(z) = \begin{cases} 5[(-25-z)/20]^4, & z \leq -25 \\ 5[(z-25)/20]^4, & z \geq 25 \\ 0 & \text{otherwise,} \end{cases} \quad (10b)$$

$$V_R(R) = \begin{cases} 0.05[(R-20)/10]^4, & R \geq 20 \\ 0 & \text{otherwise.} \end{cases} \quad (10c)$$

The number of time steps per optical cycle is 1100. For each time step the norm of the total wave function within  $0.05 \leq R \leq 20$ ,  $-25 \leq z \leq 25$  is calculated. The outgoing current flux through the borders  $z_{vd} = \pm 25$ ,  $R_{vd} = 20$  of this inner part of the grid is used to obtain differential and integrated data on the fragmentation process. If we write the outgoing wave packet in terms of a real amplitude  $A$  and a real phase  $\phi$ ,

$$\Psi(z, R, t) = A(z, R, t) \exp[i\phi(z, R, t)], \quad (11)$$

we find the current density

$$\mathbf{j}(z, R, t) = \frac{|A(z, R, t)|^2}{m} \nabla \phi(z, R, t). \quad (12)$$

Here,  $m$  is the mass of the particle. The dissociation yield  $D$  into the fragmentation channel (1) is given by the time-integrated nuclear flux in the  $R$  direction at  $R_{vd}$ ,

$$D(t) = \int_0^t dt' \int_{-z_{vd}}^{+z_{vd}} dz j_R(z, R_{vd}, t'). \quad (13)$$

Removal of the electron leads to CE, the yield of which is given by the time-integrated electronic flux in the  $z$  direction at  $z = \pm z_{vd}$ ,

$$C(t) = \int_0^t dt' \int_{0.05}^{R_{vd}} dR j_z(z_{vd}, R, t'). \quad (14)$$

We derive the nuclear momentum distribution for dissociation using the “virtual detector” method [26] and imagine virtual detectors for the nuclear and electronic flux at the positions  $R_{vd}$  and  $\pm z_{vd}$  indicated above. At each time step  $t_i$ , we numerically calculate the gradient of  $\phi$  at  $R_{vd}$  using a five-point formula for the momentum,

$$p_R^{(D)}(z, t_i) = \frac{\partial}{\partial R} \Big|_{R_{vd}} \phi(z, R, t_i). \quad (15)$$

Integration over  $z$  and binning of the momentum values for all times  $t_i$  yield the momentum distribution for dissociation.

The CE momentum spectra are obtained by first computing the ionization rate as the outgoing electron flux through  $\pm z_{vd}$  at a given internuclear distance  $R$ . During the (fast) ionization of the electron we treat the (slow) nuclei as “frozen.” For each  $R$ , the Coulomb energy  $1/R$  is released. In order to take the initial (dissociative) momentum

$$p_{\text{init}}(R, t_i) = \frac{\partial}{\partial R} \Big|_{z_{vd}} \phi(z, R, t_i) \quad (16)$$

of the nuclei into account, we add the Coulomb energy and the initial kinetic energy of the nuclei at the ionization time and obtain the final momentum

$$p_R^{(\text{CE})}(R, t_i) = \sqrt{p_{\text{init}}^2(R, t_i) + 2\mu/R} \quad (17)$$

of the released fragments. Here,  $\mu$  is the reduced mass of the nuclei. Analogous to the dissociation, integration over  $R$  and binning of the nuclear momentum values for all times  $t_i$  give the momentum distribution for CE.

## III. RESULTS

Before we discuss the vibrational-state dependence of the fragmentation of  $\text{H}_2^+$  in a 25-fs laser pulse, we consider the intensity dependence for two selected vibrational states  $|v=3\rangle$  and  $|v=6\rangle$ . The main difference between these states is the fact that  $|v=3\rangle$  lies below the threshold for the  $1\omega$

Floquet channel whereas states  $|v=5\rangle$  or higher can decay into the  $1\omega$  as well as into the  $2\omega$  [29] channel that is open for all  $v$  states (see Fig. 1). Thus, lower  $v$  initial states need excited-state contributions  $|v\geq 5\rangle$  in order to decay into the  $1\omega$  dissociation channel.

Vibrational excitation is likely in sufficiently short and intense laser pulses if the electric-field envelope changes significantly on the time scale of the vibrational motion. In this case, the time dependence of the dressed (Floquet) potential curves leads to nonadiabatic transitions between vibrational states. The influence of the pulse duration on the kinetic-energy spectra has been demonstrated by Frasiniski *et al.* [18]. We note that these transitions are mediated by the interaction of the electron cloud with the laser field, since the nuclei cannot absorb photons directly. For all calculations, we used  $\sin^2$  shaped 25-fs full width at half maximum pulses of 800-nm wavelength.

### A. Fragmentation of $H_2^+$ ( $v=3$ ) in a 25-fs laser pulse: Intensity dependence

Figure 3(a) shows the time-dependent norm

$$N(t) = \int_{0.05}^{R_{vd}} dR \int_{-z_{vd}}^{+z_{vd}} dz |\Psi(z, R, t)|^2 \quad (18)$$

and the probabilities for dissociation  $D(t)$  and Coulomb explosion  $C(t)$  for  $H_2^+$  ( $v=3$ ) interacting with a 25-fs laser pulse of intensity  $I=0.2$  PW/cm $^2$ . We find 20% probability for dissociation and 70% for CE. The time evolution of the nuclear probability density

$$P(R, t) = \int_{-z_{vd}}^{+z_{vd}} dz |\Psi(z, R, t)|^2 \quad (19)$$

is illustrated in Fig. 3(b) with contour lines indicating the electronic ionization rate due to CREI. In the density plot, the dissociation appears as a threefold jetlike structure. This structure can be associated with distinct peaks in the dissociation kinetic-energy spectrum shown as a solid line in Fig. 3(c) for intensities in the range from 0.05 to 0.5 PW/cm $^2$ . The kinetic-energy range for one- and two-photon absorption into Floquet channels for all vibrational states is given by the shaded areas. For 0.1 PW/cm $^2$ , we find the strongest dissociation. With increasing intensities, Coulomb explosion (dashed line) becomes dominant and results in the reduction of the final dissociation probability. At 0.2 PW/cm $^2$ , the CE spectrum forms a broad peak between 4 and 8 eV corresponding to internuclear distances in the CREI region in Fig. 3(b) at  $3.5 \leq R \leq 7$ . The CE peak is shifted towards higher kinetic energies for 0.5 PW/cm $^2$ , showing a substructure which reflects the nodes of the initial wave function.

### B. Fragmentation of $H_2^+$ ( $v=6$ ) in a 25-fs laser pulse: Intensity dependence

Figure 4 shows the fragmentation dynamics of the  $|v=6\rangle$  initial state. Compared to  $v=3$ , at 0.2 PW/cm $^2$  the dis-

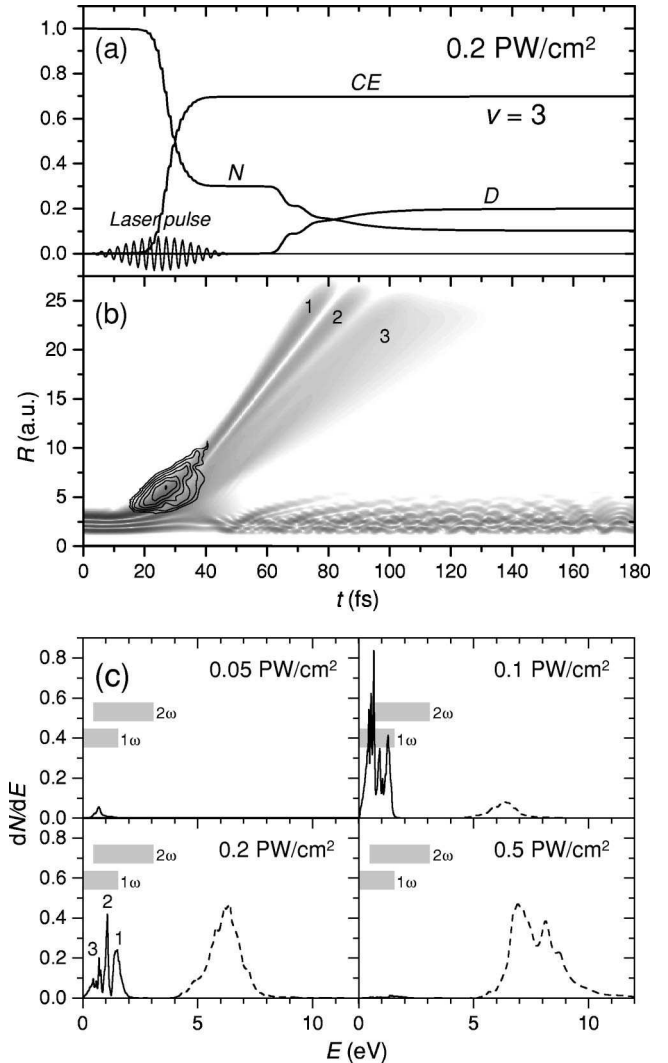


FIG. 3. Fragmentation of  $H_2^+$  ( $v=3$ ) in a 25-fs laser pulse. (a) Time-dependent norm ( $N$ ) and probabilities for Coulomb explosion (CE) and dissociation ( $D$ ) at  $I=0.2$  PW/cm $^2$ . (b) Corresponding probability density  $P(R, t)$  (logarithmic gray scale) and ionization rates (contour lines). (c) Kinetic-energy spectra for dissociation (solid lines) and CE (dashed lines) for intensities in the range from 0.05 to 0.5 PW/cm $^2$ .

sociation probability is reduced to 10% and the CE enhanced to 88%. For the lower intensities, dissociation is much stronger than for the  $|v=3\rangle$  initial state, mainly due to the opening of the  $1\omega$  channel [note the different scale in the upper row of Fig. 4(c)]. The CE spectra behave similarly to the  $v=3$  case but with a generally higher ionization probability. The intensity dependence of the fragmentation (dissociation and CE) probabilities for the  $|v=3\rangle$  and  $|v=6\rangle$  initial states is summarized in Fig. 5. For  $|v=3\rangle$ , the dissociation channel shows a maximum of 37% around 0.1 PW/cm $^2$  whereas for  $|v=6\rangle$  the dissociation probability increases further with decreasing intensity up to 73% at 0.05 PW/cm $^2$ . The ionization (CE) probability reaches saturation for both initial states at 0.5 PW/cm $^2$ .

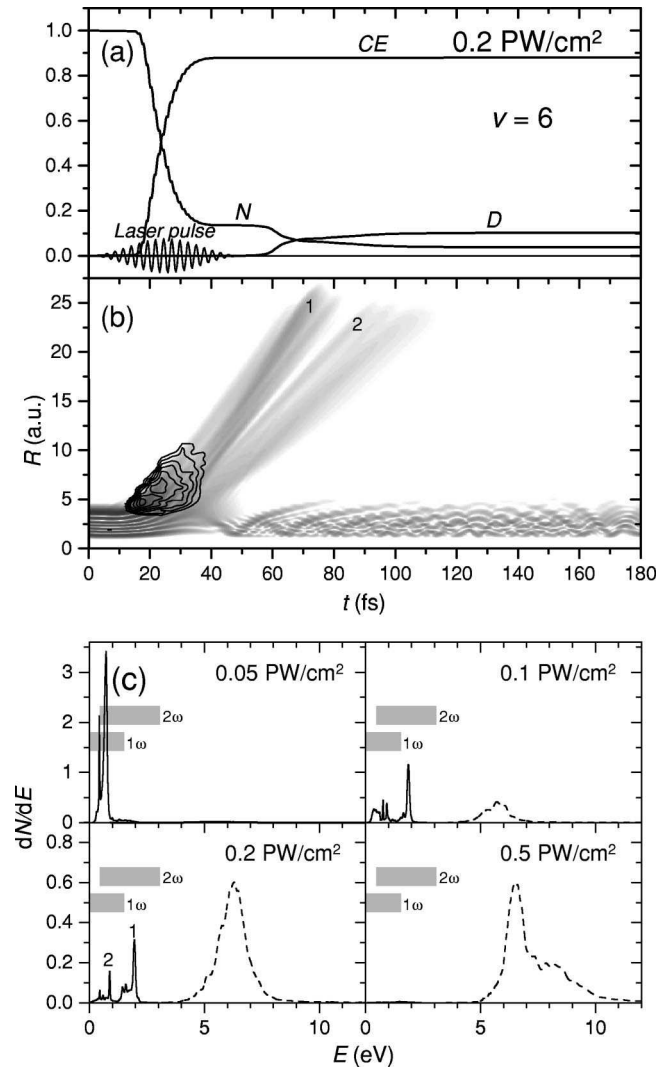


FIG. 4. Same as Fig. 3 but with  $|v=6\rangle$  as initial state.

### C. Fragmentation of $H_2^+$ in a 25-fs 0.2-PW/cm<sup>2</sup> laser pulse: Vibrational-state dependence

As mentioned above, for 25-fs pulses we have a competition of dissociation and CREI at an intensity of  $\approx 0.2$  PW/cm<sup>2</sup>. Figure 6 shows the fragment kinetic-energy spectra for the initial states  $|v\rangle$ ,  $v=0-2, 4, 5, 7-9$  [for  $v=3, 6$ , see Figs. 3(c) and 4(c)]. These states significantly contribute to a Franck-Condon distribution one would expect for  $H_2^+$  created in an ion source [24] (see Franck-Condon factors in Fig. 7). We find the vibrational ground state being almost stable against fragmentation. With increasing  $v$ , first dissociation dominates with a maximum probability of 33% for  $|v=2\rangle$ . Then CREI becomes more and more favorable at the expense of dissociation with ionization probabilities  $>90\%$  for  $|v=7\rangle$  and higher (see Fig. 7). There is also a noticeable shift in the dissociation spectra from the  $1\omega$  to the  $2\omega$  region (indicated by the shaded areas).

The  $v$ -dependent behavior of the ionization probability can be explained in the following way. For the low  $v$  states, we have first an expansion of the molecule due to bond softening. Once  $R$  reaches the “critical” region of 5–10, CREI

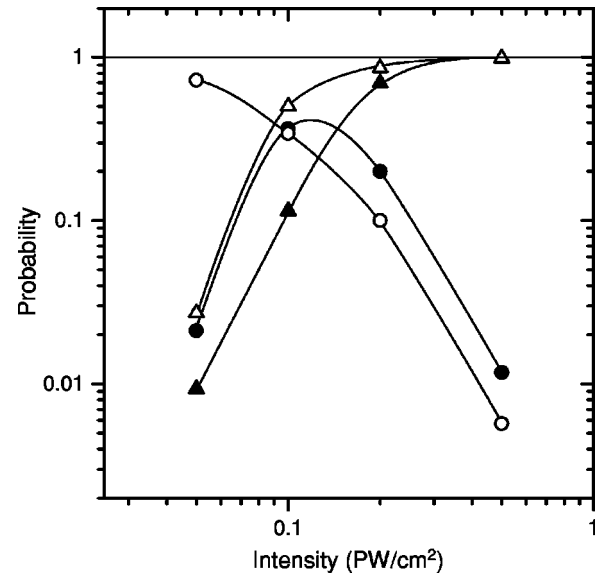


FIG. 5. Intensity dependence of the fragmentation probabilities.  $|v=3\rangle$  initial state:  $\bullet$ , dissociation;  $\blacktriangle$ , CE.  $|v=6\rangle$  initial state:  $\circ$ , dissociation;  $\triangle$ , CE. The curves joining the points only serve to guide the eye.

becomes likely. With increasing  $v$  the overlap of the initial wave function with the CREI region becomes larger. In addition, these states are more sensitive to the bond softening. The increasing overlap of the initial states with the CREI

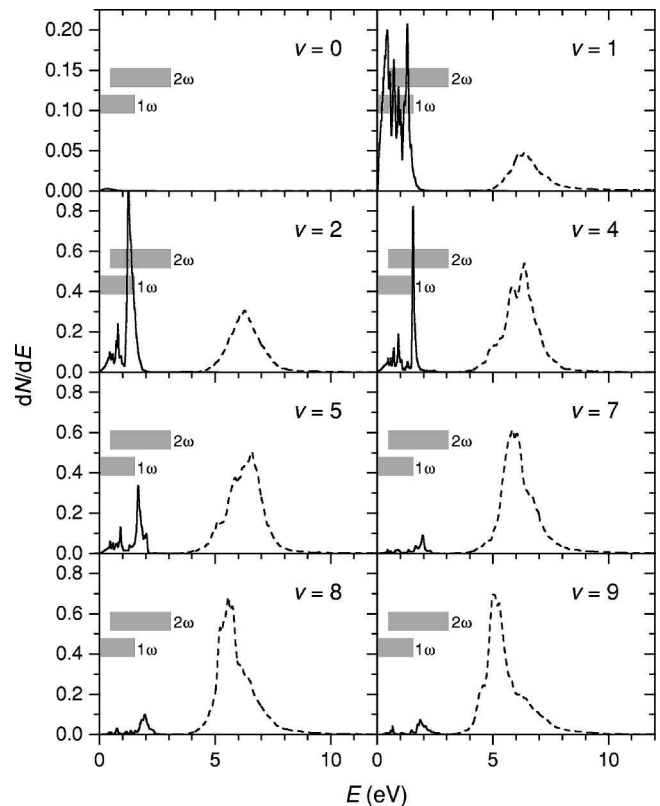


FIG. 6. Fragmentation of  $H_2^+$  in a 25-fs laser pulse at  $I = 0.2$  PW/cm<sup>2</sup>. Kinetic-energy spectra for dissociation (solid lines) and CE (dashed lines) for initial vibrational states  $|v\rangle$ ,  $v=0-2, 4, 5, 7-9$  [for  $v=3, 6$  see Figs. 3(c) and 4(c)].

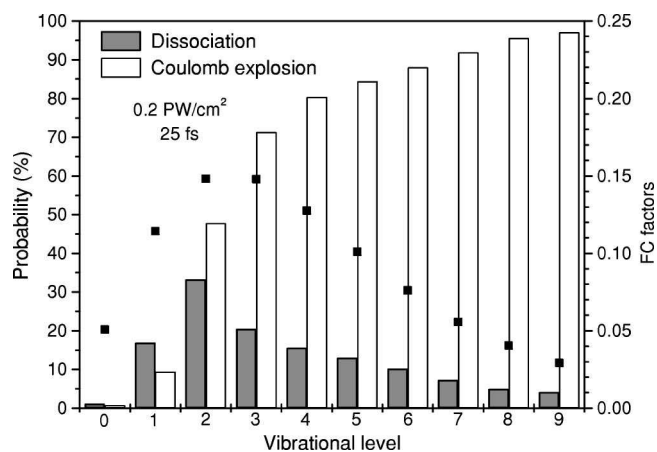


FIG. 7. Fragmentation of  $\text{H}_2^+$  in a 25-fs laser pulse at  $I = 0.2 \text{ PW/cm}^2$ . Initial-vibrational-state dependence of the probabilities for dissociation and CE. The black squares show the Franck-Condon factors (right scale)  $|\langle \text{H}_2^+, v | \text{H}_2, v=0 \rangle|^2$ .

region shifts the range where ionization occurs towards larger  $R$ . This results in lower Coulomb energy release, i.e., in a slight shift of the CREI peak in the spectrum from 6.5 eV ( $v=1$ ) to 5 eV ( $v=9$ ).

So far no experiment with selected pure initial vibrational states has been done due to technical difficulties. Ion sources usually provide a broad distribution of  $|v\rangle$  states. The kinetic-energy spectrum for this kind of ensemble is shown in Fig. 8 for a 25-fs, 0.2-PW/cm<sup>2</sup> pulse. Here, we added the spectra for all significant states ( $v=0, \dots, 9$ ), weighted with the Franck-Condon factors. As a result of this summation, much of the interesting  $v$  dependence is lost. An alternative to the preparation of pure states is the use of not too intense, “long” ( $>100$  fs) pulses, which avoid nonadiabatic vibrational excitations. In this case, high-resolution spectroscopy allows the identification of individual  $|v\rangle$  states in the  $1\omega$  and  $2\omega$  dissociation spectrum, as it has been done in the recent experiment by Sändig, Figger, and Hänsch [23].

#### IV. CONCLUSION

Using wave-packet propagation calculations within a collinear reduced-dimensionality model, we investigated the fragmentation of the  $\text{H}_2^+$  molecular ion in 25-fs, 800-nm laser pulses in the intensity range 0.05–0.5 PW/cm<sup>2</sup>. Our model represents both the nuclear and electronic motion by one degree of freedom including non-Born-Oppenheimer couplings. By introduction of a modified soft-core Coulomb potential for the electron-nucleus interaction with a softening function that depends on the internuclear distance, we were able to reproduce accurately essential properties of the

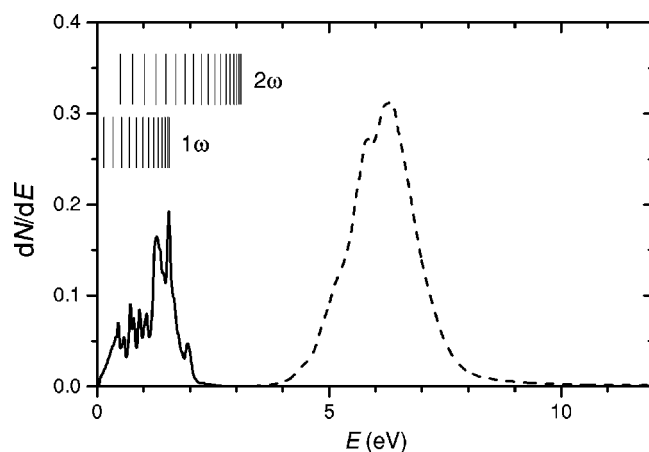


FIG. 8. Fragmentation of  $\text{H}_2^+$  in a 25-fs laser pulse at  $I = 0.2 \text{ PW/cm}^2$ . Kinetic-energy spectra for dissociation (solid lines) and CE (dashed lines) for a statistical (Franck-Condon) mixture of initial vibrational states.

“real” 3D molecule. We calculated the probabilities and kinetic-energy spectra for dissociation and CE of the molecule for various intensities and initial vibrational states. For the initial vibrational states  $|v=3\rangle$  and  $|v=6\rangle$  and different intensities, we demonstrated the effect of the threshold ( $v \geq 5$ ) for the  $1\omega$  Floquet channel. For 0.2 PW/cm<sup>2</sup>, our calculation revealed a strong vibrational-state dependence of the branching ratio between dissociation and CE. We found that dissociation dominates for molecular ions that are prepared in the two lowest vibrational states only, while CE becomes more and more dominant for higher vibrational states. The latter is caused by the increasing overlap of the initial nuclear wave function with the region of internuclear distances where CREI leads to high ionization probabilities. We also considered an incoherent (Franck-Condon) mixture of vibrational states, e.g., a beam of molecular ions created in an ion source. In contrast to a coherent superposition of vibrational states, the time dependence of which could be studied in a two-pulse pump-probe experiment [6], some of the interesting  $v$  dependence is lost after averaging over the Franck-Condon distribution. We hope that our work stimulates challenging future experimental investigations including the preparation of selected vibrational states.

#### ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation (Grant No. PHY-0071035) and the Division of Chemical Sciences, Office of Basic Energy Sciences, Office of Energy Research, U.S. Department of Energy. B.F. acknowledges the financial support from the Deutsche Forschungsgemeinschaft.

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- [29] We note that this channel corresponds to a *net* absorption of two photons. In fact, this channel exhibits first an absorption of three photons at the “ $3\omega$ ” crossing followed by a reemission of one photon at the subsequent “ $1\omega$ ” crossing (see also Fig. 1).