

Measurement of Planck's constant through Black Body Radiation Analysis

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The experiment used a black body approach to experimentally measure Planck's reduced constant, h by using the Planck Radiation Law, which relates the intensity and wavelength of light to the temperature of the emitting black body. The measurement was accomplished by using a simplified detection system involving a photodiode and diffraction grating to measure the intensity of emitted light as a function of the wavelength. From the experiment it was found that the value of h was $2.8 \pm 1.5 \times 10^{-35}$ Js.

1 Introduction

1.1 Background

The purpose of the experiment was to measure Planck's reduced constant, h , through the observation of black body radiation. Planck's constant is a fundamental constant that describes the scales of quantum mechanical systems. Planck first derived the constant during his formulation of black-body radiation by considering that light itself is quantized in what is now known as photons¹.

There are many different methods to measure Planck's constant as much of atomic physics is founded upon the quantum theory and therefore Planck's constant. Such classic experiments, such as the measurement of Compton scattering, the photoelectric effect, the measurement of the emission lines of hydrogen as well as black body radiation all have working model's that contain a method in which to experimentally validate the value of h . In the experiment performed, black-body radiation was used to measure h as it only requires minimal available equipment to measure h and calculate its value². However, the simplicity of experimental design comes at the cost of

reduced precision, which will be addressed later in the paper.

1.2 Theory

The working theory behind the experiment involves the treatment quantization of light together with the understanding of statistical physics. According to Planck's law, an object will radiate a spectrum of light dependent on the temperature of the object. The Planck theory proposed that the energy carried away from the object in the form of photons with an energy given by:

$$E = \hbar\omega \quad (1)$$

Where \hbar is Planck's reduced constant and ω is the angular frequency of the photon emitted. If emitted photons are treated as a photon gas, one can use a statistical approach to derive the 'Planck radiation law'³, which is given as follows:

$$I = \frac{\hbar\omega^3}{\pi^2 c^3 \text{Exp}\left(\frac{\hbar\omega}{k_b T}\right) - 1} \quad (2)$$

Where I is the intensity of the emitted light, T is the temperature of the black body emitter, ω is the frequency of the photon and k_b is Boltzmann's constant. It is important to note that Equation 2 represents the intensity as a function of frequency, and therefore by relation the

¹ "Thermal Physics". C. Kittel, H. Kroemer. Pg 91.

² "Minimal Apparatus for determination of Planck's Constant," R.E. Crandall J.F. Delord, 1983.

³ "Thermal Physics". C. Kittel, H. Kroemer. Pg 94.

wavelength, of the light. Furthermore the dependence upon the temperature of the black body implies that the number of photons a black body emits at a specific frequency is determined by the temperature of the black body. It is this relationship that is exploited in order to experimentally determine Planck's constant.

Consider the ratio of two separate intensities measured at the same frequency, I_1 and I_2 . From Equation 2, this ratio is given as:

$$\frac{I_1}{I_2} = \frac{\text{Exp}\left(\frac{\hbar\omega}{k_b T_2}\right) - 1}{\text{Exp}\left(\frac{\hbar\omega}{k_b T_1}\right) - 1} \approx \frac{\text{Exp}\left(\frac{\hbar\omega}{k_b T_2}\right)}{\text{Exp}\left(\frac{\hbar\omega}{k_b T_1}\right)} \quad (3)$$

The approximation of the formula is valid in the regime in which $\omega > 6.5 \times 10^{11} \text{ T}$ where ω is given in units of Hertz and T in units of Kelvin⁴. For the visible spectrum of light as well as the near infrared spectrum, these conditions are well satisfied. Consequently, Equation 3 can be expressed to give Planck's reduced constant:

$$\hbar = \frac{k_b}{\omega \left(\frac{1}{T_2} - \frac{1}{T_1}\right)} \text{Ln}\left(\frac{I_1}{I_2}\right) \quad (4)$$

From Equation 4, \hbar can be experimentally determined by the ratio of intensities of a black body at two separate temperatures. However in the case of the experiment, it is more useful to collect a multitude of data relating the intensity to multiple values of temperature for a specific wavelength of light. In this scenario, it is more precise to use the method of least-squares to extract the value of Planck's constant. In linear form, Equation 4 becomes:

$$\frac{1}{T} = \left[\frac{k_b}{\hbar\omega} \text{Ln}(I_0) + \frac{1}{T_0} \right] + \frac{-k_b}{\hbar\omega} \text{Ln}(I) \quad (5)$$

Where $\frac{1}{T}$ is the dependent variable and $\text{Ln}(I)$ acts as the independent variable. Using this form, a least-squares analysis will yield \hbar in relationship to the selected frequency at a given temperature.

2 Experimental Methods

For the experiment, a tungsten filament light bulb was used as the emitting source. The filament, in conjunction with an apparatus that could record the relative intensity of light at various frequencies, Equation 5 could be used to relate the frequencies, intensities and temperatures to determine Planck's reduced constant. The apparatus used contained a tungsten filament light as the black body emitter, a diffraction grating to separate the various wavelengths of light and a photodiode to detect relative intensities of incident light. A diagram of the apparatus is given below in Figure 1:

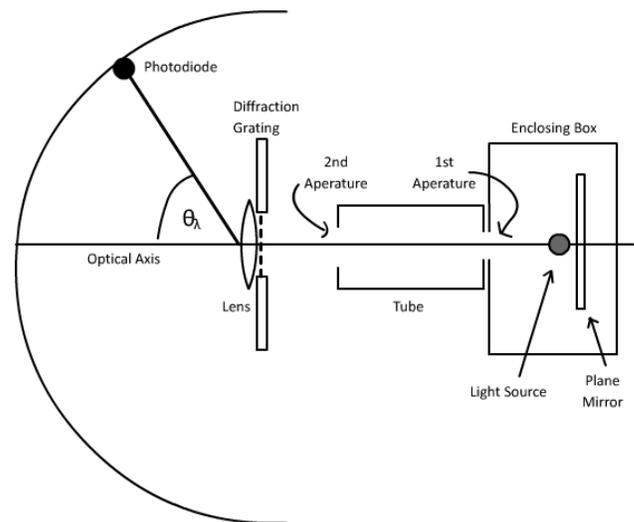


Figure 1 - Diagram of the apparatus used

The light from the black-body source is put through a system of apertures in order to focus the light beam. The first aperture is a 1 cm circular hole in a box the completely surrounds the light source. This reduces a large amount of background light as well as focuses a narrower beam of light towards the diffraction grating. Next a 15 cm long tube with a 2.5cm by 2.5cm aperture was placed next to the first aperture so as to block out more background light and ensured only beams of light from the source on a near parallel path with the optical axis were passed through to the diffraction grating. The beam of light is then passed through a diffraction grating and lens system that causes the different wavelengths of light to separate with an angular dependence given by:

$$m\lambda = d(\text{Sin}\theta_\lambda - \text{Sin}\theta_i) \quad (6)$$

⁴ "Planck's Constant Determination From Black-Body Radiation", J. Dryzek 1991

Where d is the width of a line in the diffraction grating, θ_λ is the angle from the optical axis that the photodetector is placed, θ_i is the angle of the incident beam upon the diffraction grating and m is the order of diffraction which in the case of the experiment is 1. The detector is setup on a pivot so that its angular location with respect to the diffraction grating can be easily measured and Equation 6 applied. Note for the experiment performed, the value of d was determined using a Helium-Neon laser with a 633nm wavelength as a calibrated source and measuring the variance on the angle caused by the diffraction grating. From the measurements, it was found that $d = 1870 \pm 30$ nm. It was also assumed that $\theta_i = 0^\circ \pm 2^\circ$ to account for any small variances due to the diffraction grating not being perfectly perpendicular to the optical axis.

The temperature of the filament in the light bulb was controlled by the voltage setting on a variable AC voltage source. For temperatures of approximately 2500K, tungsten obeys a linear relation between the resistance of the filament and its temperature, given as:

$$R(T) = R_0[1 + \alpha(T - T_0)] \quad (7)$$

Where R_0 represents the resistance at a temperature T_0 and α is the temperature coefficient for tungsten. The tabulated value of α for tungsten is $0.0045 \Omega/K$, however due to material impurities the actual value for the filament could be different for the tabulated value, thus an uncertainty of $0.0005 \Omega/K$ was considered for α as such a deviation allows α to fall within range of other common conducting metals such as copper, aluminum, nickel and silver. Thus if there were any other metals contained within the wire, the uncertainty in the value of α includes such factors.

The photodiode used was connected in the following circuit as to measure the relative intensities:

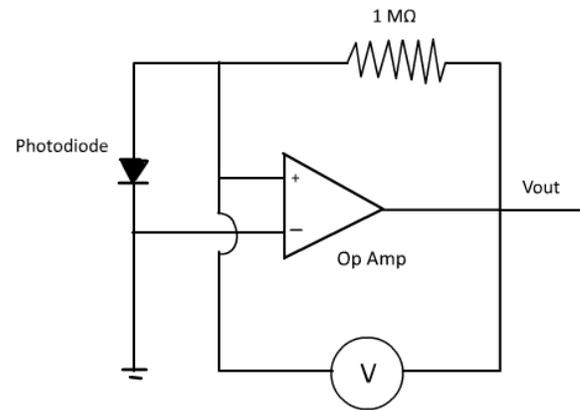


Figure 2 - Diagram of the photodiode detection circuit. The Op Amp was powered by $\pm 12V$ leads

The op amp and resistor combination helped to increase the voltage measured from the photodiode so that was within the range of the voltmeters used. An important aspect of the photodiode used is that it follows a responsivity curve, meaning that the amount of current that is produced per photon on the photodiode is dependent on the wavelength. To account for this, the photodiode has been calibrated for a responsivity curve that relates the relative intensity to wavelength. Thus in the case of the experiment the measured voltage acts as measurement of intensity⁶. Taking into account of the responsivity as a voltage gain, the intensity voltage is given as⁷:

$$V_{\text{int}} = R(\lambda)V_{\text{meas.}} \quad (8)$$

Where V_{int} , $V_{\text{meas.}}$ are the weighted and unweighted measurements of the voltage respectively. To simplify calculations, the function $R(\lambda)$ can be approximated from data points in the following table⁸:

⁵ "Planck's Constant Determination From Black-Body Radiation", J. Dryzek 1991

⁶ Note that in the experiment, $V \propto I$

⁷ "FDS100 Specifications Sheet", Thor Labs. 2006.

⁸ $R(\lambda) \approx -2.8 \times 10^{-12}\lambda^5 + 8.5 \times 10^{-9}\lambda^4 - 9.6 \times 10^{-6}\lambda^3 + 5.0 \times 10^{-3}\lambda^2 - 0.1\lambda + 5.725$ from regressions

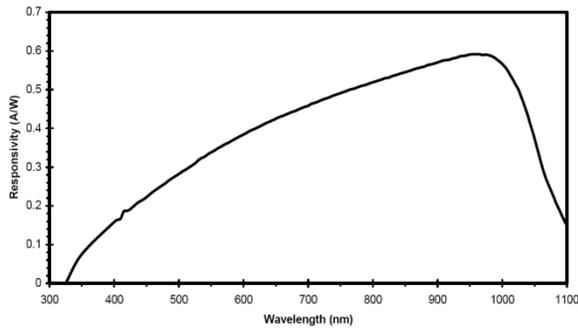


Figure 3 - Responsivity of the FDS100 Photodiode. From the response curve it can be seen the diode has a limited range of wavelengths it can detect adequately

For the experiment, there were a variety of uncertainties that were involved that directly affect the results. Below in Table 1 is a summary of the uncertainty values of various experimental parameters as well as any constants measured for the experiment:

Table 1 - Experimental parameters and error values

Quantity	Value/Error
Input Voltage	0.2 V
Input Current	0.01 A
Photodiode Voltage	10 mV
θ_λ , Detector Angle	1.7 Degrees
R_0 , Init. Resis.	45 ± 1 Ohms
α , Temp. Coeff.	0.0045 ± 0.005 ΩK^{-1}
T_0 , Room Temp.	293 ± 1 K

Note that the error in the detector angle is caused by the width of the photo detector primarily, as its width of 5.8 mm at a distance of 19.5 cm has a corresponding angle of 1.7 DEGREE. This angular uncertainty is an order of magnitude greater than that of the uncertainty cause by diffraction grating, thus it is used as the uncertainty for the angle measurement. The error of the voltmeters and ammeters covers the average fluctuations of the experimental value during measurement. These values are used with equations 5, 6, 7 and 8 along with their error propagation counterparts to determine the intensity as a

function of temperature and wavelength and their associated error⁹.

3 Experimental Observations

3.1 Black Body Curves

For the data set, the points of data relating the intensity incident upon the photodiode and the angle of the photodiode with respect to the diffraction grating were measured. The following is a summary plot of the results:

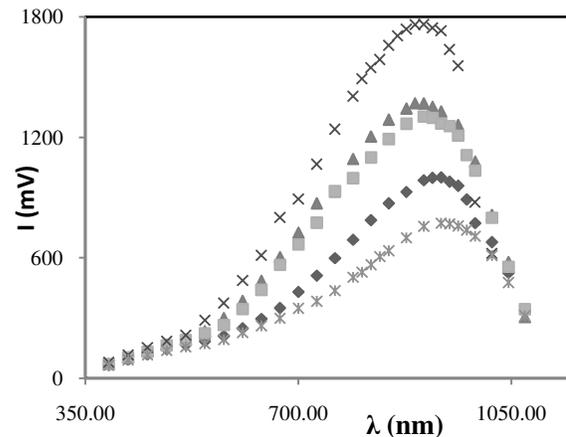


Figure 4 - Plot the intensity versus the wavelength of incident light. The highest peak and successive lowering of peaks represent black bodies at 3300K, 3250K, 3200K, 3100K, and 3050K respectively.

Note that the intensity plotted is voltage produced by the photodiode as well taking into account as the corresponding responsivity to each wavelength. From these curves it is apparent that the data visually corresponds to the spectral output intensity as predicted by Planck's law in Equation 2. It is important to note that entire black body curves in the near infrared regimes cannot be mapped very easily due to the photodiode used. The responsivity curve of the photodiode only has non-zero values between approximately 300 to 1100 nm which prohibits measurements of wavelengths not in the range. Secondly from Equation 6 for the diffraction angle, it can be found for the experimental setup that the second order diffractions of the light begin to occur around 25 degrees

⁹ The error propagation formulas Equations 5, 6, 7, and 8 and their corresponding error propagation derivations are omitted from the section for simplicity's sake.

(approximately 800nm in the first order) which causes errors in the intensity readings in the higher first order measurements.

3.2 Planck’s Constant

In order to experimentally measure Planck’s constant, three specific wavelengths of study were chosen. These wavelengths were chosen to correspond to the 15, 20 and 25 degree angles between the photodiode and the diffraction grating. These wavelengths represent a blue, a red and a near infrared color, and each lies at a different portion of the black body power spectrum curve. The multiplicity of wavelengths studied allowed a more varied approach to determining h . For each wavelength, the voltage of the light bulb, therefore the temperature from Equation 7 was gradually increased in five volt increments. At each temperature change, the relative intensity of the light as reported from the photodiode was recorded for each of the three wavelengths. A summary of the data collected is displayed below in Table 3:

Table 2 - Data comparing the input AC voltage (in V), Temperature (in K) to the recorded intensities at the photodiode at three separate wavelengths. Equation 7 was used to calculate the temperature at each voltage setting

		λ (nm)		
Voltage (V, $\sigma=0.5$)	Temp (K)	480 \pm 83	640 \pm 81	790 \pm 78
		Intensity (mV)		
70	2400 \pm 100	140 \pm 52	250 \pm 50	420 \pm 77
75	2400 \pm 100	150 \pm 55	280 \pm 57	490 \pm 89
80	2400 \pm 100	150 \pm 57	320 \pm 65	520 \pm 95
85	2500 \pm 100	160 \pm 61	360 \pm 73	600 \pm 110
90	2500 \pm 100	170 \pm 64	400 \pm 82	670 \pm 120
95	2600 \pm 100	180 \pm 67	460 \pm 94	760 \pm 140
100	2700 \pm 100	190 \pm 71	510 \pm 100	830 \pm 150
105	2700 \pm 100	210 \pm 79	530 \pm 110	940 \pm 170
110	2800 \pm 110	230 \pm 85	630 \pm 130	1000 \pm 190
115	2800 \pm 110	240 \pm 88	700 \pm 140	1100 \pm 210
120	2900 \pm 110	250 \pm 92	760 \pm 150	1200 \pm 220
125	2900 \pm 110	250 \pm 95	820 \pm 170	1300 \pm 240

By performing linear least-squares fit of $1/T$ versus $\ln(I)$ in Equation 5, it can be seen by comparison that for a linear fit of $y = bx + a$ that the corresponding values of a and b are:

$$a = \frac{k_b}{h\omega} \ln(I_0) + \frac{1}{T_0} \tag{9}$$

$$b = \frac{-k_b}{h\omega} \tag{10}$$

Where it was taken that I_0 and T_0 were the values of the lowest temperature, in this case the data for when the voltage was set to 70 volts. Therefore solving Equations 9 and 10 will give the experimentally measured values of h . The following are the linear plots of $1/T$ versus $\ln(I)$:

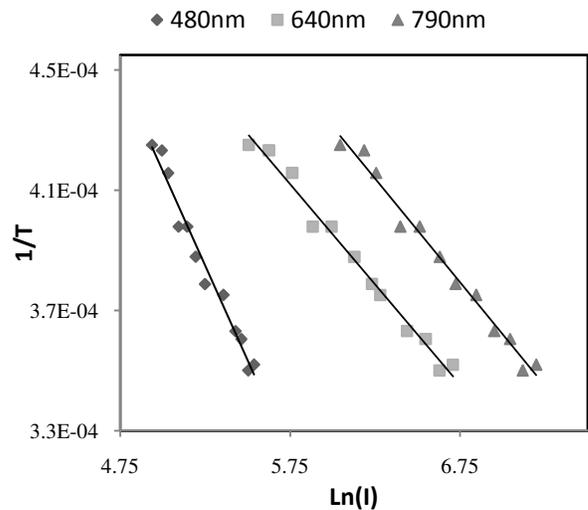


Figure 5- Plot of $1/T$ vs. $\ln(I)$ for the three wavelengths with corresponding linear fits. From left to right: 480nm data, 640nm data, 790nm data

The plots in Figure 5 show a linear relationship, suggesting that Equation 5 satisfies the approximations made during the theory.

The following data gives the values of the least squares fits for a and b and the corresponding values for h as derived from Equations 9 and 10.

Table 3 – Data for the linear fit of results of Table 2 and the plots of Figure 5. The a and b values correspond to those listed in Equation 9 and 10 respectively. The calculated values of each \hbar by solving equations 9 and 10 are also listed.

λ (nm)	a (1/K 10-4)	b (1/K 10-5)	\hbar_a (Js 10 ⁻³⁵)	\hbar_b (Js 10 ⁻³⁵)
480 ± 11	10 ± 3.0	-13 ± 6.5	2.8±1.6	2.8±1.5
640 ± 12	7.9 ± 2.1	-6.7 ± 3.4	6.9±4.0	7.0±3.6
790 ± 14	8.4 ± 2.4	-6.9 ± 3.5	8.3±4.7	8.4±4.3

4 Discussion

4.1 Results

From Table 3, the calculated values of \hbar found in section 3.2 were found to be only accurate up to an order of magnitude at best compared to the standardized value of $1.0545 \times 10^{-34} \text{ Js}$ ¹⁰. The result is nothing less than depressing as no true precision can be assigned to the value of \hbar beyond its order of magnitude of 10^{-35} Js . There are many factors that could have caused the result to have been heavy in error. First of all, the temperature dependence of the resistance in Equation 7 was assumed to be linear. As stated, this approximation is valid up to around 2500K. However, in Table 2 it can be seen for most data points, the temperature of the wire exceeds 2500K, by as much as 400K. Thus this would change the result significantly as the value of \hbar is derived from data involving T, specifically $1/T$.

Another interesting source of error that is shown is the detection of the photodiode at higher wavelengths, specifically in the near infrared range. As mentioned, at around 800nm, the second order diffractions of the light begin to overlap with the first order diffractions, causing a false reading of the intensity measured. Notice that this error shows itself in the results; the calculated values of \hbar in Table 3 become less precise and less accurate as the wavelength is increased. A priori it was assumed that the high wavelength values would result in more accurate values of \hbar because the intensity was larger, thus making the measurement upon the photodiode more precise the

photodiode responded better at such frequencies. However due to the second order effect and other error, the low intensity, short wavelength light yielded more accurate results.

4.2 Conclusion

The measurement of Planck's reduced constant by observing the light emitted from a black body was found to have large uncertainties due to multiple error propagations in the experiment. No doubt with a more precise setup, such as a smaller photodetector area or more precise voltmeters, the uncertainty would be reduced. However, calculations show that reducing all uncertainties by two orders of magnitude only yield a single order of magnitude decrease in the uncertainty of \hbar . Thus a method to directly measure the value of \hbar , such as the photoelectric effect with a single working equation, would provide a much more direct method of experimental measurement because it has less factors for error.

Within a similar experiment¹¹, color filters were used instead of a diffraction grating to select specific wavelengths of the emitted light for measurements. By using such methods, as well as using an optical pyrometer to measure the temperature of the filament and a vacuum photocell to measure the intensity. With the more complex set, values of \hbar accurate up to 2% were obtained. Therefore it is possible to measure \hbar through black-body methods, however it requires a more complex setup than was employed in the experiment performed.

5 Acknowledgements

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¹⁰ 2006 NIST standardized value for \hbar

¹¹ "Planck's Constant Determination From Black-Body Radiation", J. Dryzek 1991