12: 17, 20, 25, 52, 33, 44, 47

12.17: Done in notes

12.20: (a) (i) $A = (15, 5, 3, 0)$
$B = (5, 10, 8, 0)$
$\Delta\vec{x} = (+10, +5, +5, 0)$
$I = (-10^2 + 25 + 25) = -50$ timelike

(ii) No

(iii) Yes! $v = \hat{x}(+.5c) + \hat{y}(+.5c)$
$\therefore (\delta\bar{x}) = \gamma(\Delta x - v_x \Delta t) = 0,$
$v_x = \Delta x/\Delta t = +.5c$, then
do similar transf. for y.

(b) $A = (1, 2, 0, 0)$
$B = (3, 5, 0, 0)$
$\Delta\vec{x} = (2, 3, 0, 0)$, $I = -4 + 9 > 0$, spacelike
ii) yes iii) No
↳ $\Delta\bar{t} = \gamma(\Delta t - v_x \Delta x/c^2)$, $v_x = \frac{\Delta t}{\Delta x} c = \frac{2}{3}c$.

12.32 

(let $c = 1$)

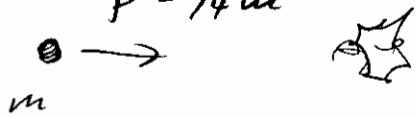
$3m = \sqrt{M^2 + p^2}$
$= M^2 + p^2$ $\quad M^2 = 9m^2 - p^2$
what is p? $\sqrt{p^2 + m^2} = 2m$ so $p^2 + m^2 = 4m^2$
or $p^2 = 3m^2$. So
$M^2 = 9m^2 - 3m^2 = 6m^2$
$M = \sqrt{6}\,m$
$\gamma = "E/mc^2" = 3m/m$
$= 3/\sqrt{6}$; $v = \left(1 - \frac{1}{\gamma^2}\right)^{1/2} = c/\sqrt{3}$

?

12.33

$E = 5/4\, m$

$p = 3/4\, m$



Mom $\quad p = p_2 - p_1$

Energy $\quad \sqrt{p^2 + m^2} = p_1 + p_2$

$2p_2 = p + \sqrt{p^2 + m^2}$

$p_2 = \frac{m}{2}\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right) = \underline{m}$

$E_2 = m(c^2)$

$E_1 = \frac{1}{4} m(c^2)$

$p_1 = -\frac{1}{2}\left(\frac{3}{4}m - p_2\right) = \underline{\frac{m}{4}}$

12.44

a)

(95) This is not as easy as it looks. | 17, 20, 25

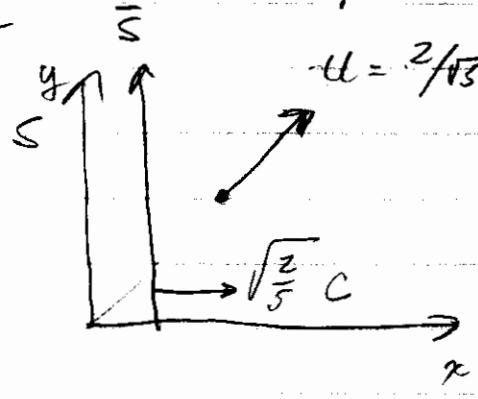
Assign 12.25

Do not do, but warn about $\vec{N}$'s.

$\vec{N}_{xyz} = \gamma \vec{u}_{xyz}$, $\vec{N}^0 = \gamma c$

Let $c = 1$

$u = 2/\sqrt{5}$



$u_x = \sqrt{2/5}$

$u_y = \sqrt{2/5}$

$\gamma_{part} = \frac{1}{\sqrt{1-\frac{4}{5}}} = \sqrt{5}$

$N_x = u_x \gamma_{part} = \sqrt{2}$

$N_y = u_y \gamma_{part} = \sqrt{2}$

$N^0 = \gamma_{part} c = \sqrt{5}$

Now view from $\bar{S}$: $\beta_{rel} = \sqrt{\frac{2}{5}}$, $\gamma_{rel} = \frac{1}{\sqrt{1-2/5}} = \sqrt{5/3}$

A. Transform ordinary vel:

$$\bar{u}_x = \frac{u_x - \beta_{rel} c}{1 - \frac{u_x \beta_{rel} c}{c^2}} = \frac{\sqrt{2/5} - \sqrt{2/5}}{1 - \sqrt{2/5}\sqrt{2/5}} = 0$$

$$\bar{u}_y = \frac{u_y}{\gamma_{rel}(1 - u_x \beta_{rel})} = \frac{\sqrt{2/5}}{\sqrt{5/3}\left(1 - \frac{2}{5}\right)} = \frac{\sqrt{\frac{2}{5}\frac{3}{5}}}{} \quad \frac{5}{3} = \frac{\sqrt{6}}{3} = \sqrt{\frac{2}{3}}$$

$= .816$ $\left(\text{so } \bar{\gamma}_{part} = \frac{1}{\sqrt{1-\frac{2}{3}}} = \sqrt{3}\right)$

Transform 4-vel:

$\bar{N}_x = \gamma_{rel}(N_x - \beta N^0) = \gamma_{rel}\left(\sqrt{2} - \sqrt{\frac{2}{5}}\sqrt{5}\right) = 0$ ✓

$\bar{N}_y = \sqrt{2}$ (check: $\stackrel{?}{=} \bar{u}_y \bar{\gamma}_{part} = \sqrt{\frac{2}{3}} \cdot \sqrt{3} = \sqrt{2}$ ✓)

$\bar{N}^0 = \gamma_{rel}(N^0 - \beta N_x) = \sqrt{\frac{5}{3}}\left(\sqrt{5} - \sqrt{\frac{2}{5}}\sqrt{2}\right) = \sqrt{\frac{5}{3}}\left(\frac{5-2}{\sqrt{5}}\right) \stackrel{?}{=} \sqrt{3}$ ✓

$\stackrel{?}{=} \sqrt{3}$? Egad!

4 5 15 16

P.S. 85

12.4 $\tfrac{1}{2}c + \tfrac{1}{3}c$ → $\tfrac{3}{4}c$

a) Galileo: $v_B = \tfrac{5}{6}c$, reaches car

b) $v_x = \dfrac{1/2 + 1/3}{1 + \frac{1}{2}\cdot\frac{1}{3}} = \dfrac{5/6}{7/6} = \dfrac{5}{7}c = .71c < .75c$

Does not reach it.

12.5.

b) Synchronized, reads noon. ("observe")

a) Clock is 90×10^{9} m away, takes ("see")

$90(9)/3(8) = 300$ sec. to get light to you,

Reads 5 min. before noon.

12.15

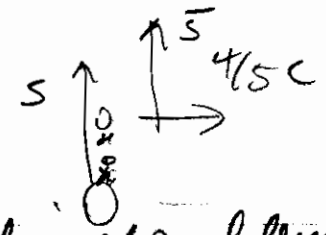
	Gr	Pol	Out	Bull	Esc.
Gr	0	1/2	3/4	5/7	✓
Pol.	1/2	0	2/5	1/3	✓
Out'l.	−3/4	−2/5	0	−1/13	✓
Bull.	−5/7	−1/3	+1/13	0	✓

units of c

anti-symm matrix.

Out'l rel pol: $\dfrac{3/4 - 1/2}{1 - 3/4\cdot 1/2} = \dfrac{2/8}{5/8} = 2/5$

Bull rel out'l $= \dfrac{-3/4 + 5/7}{1 - 3/4\cdot 5/7} = \dfrac{-(21-20)/28}{(28-15)/28} = -1/13$



12/16

$\gamma = 5/3$

Analysis of problem:

In S: Leaves at $(t,x) = (0,0)$ arrives at $(15\text{ y}, 12\text{ ly})$

In $\bar{S}$: Leaves at $(\bar{t}, \bar{x}) = 0,0$, arrives at $(9\text{ y}, 0)$

Now if she looks back at clock in S on earth, $t = \gamma(\bar{t} + v\bar{x})$, and $\bar{x} = -(4/5)(9) = -36/5$ ly.

So $t = \frac{5}{3}\left(9 - \frac{4}{5}\cdot\frac{36}{5}\right) = 5.4$ y. (this is less than 9 yr. ÷ 1). If she jumps to $\bar{S}$, however, the earth clock, which was the "leading" clock and thus reading early by $(\gamma v)\frac{36}{5} = 9.6$ yr becomes the "lagging" clock, reading late by 9.6 yr., so the "earth clock" has (appears to) advance by 19.2 yr. during the turnaround. Now the earth returns to her, with brother's clock advancing 5.4 y, so the earth clock has advanced by $\underbrace{19.2\text{ yr}}_{\text{Turnaround advance}} + \underbrace{2\times 5.4\text{ yr}}_{\text{Travel advance}}$

$= \underline{30\text{ yr.}}$ (Diff this really is thinking about this the hard way!)

answers to questions:

a) Brother sees her "slow" clock go 18 y, so his goes $18\gamma = \underline{30\text{ y}}$. He is $30 + 21 = \underline{51\text{ yrs. old}}$

b) Star is $(15\text{ y})(4/5\text{ c}) = \underline{12\text{ l.y. away}}$

c) (x,t) of jump are $(12\text{ l.y.}, 15\text{ y.})$ in S.

d) $(\bar{x},\bar{t})$ are $(\gamma(x-vt), \gamma(t-vx)) = (0, 9\text{ y})$. (We knew this!)

e) $(\tilde{x},\tilde{t})$ are $(\gamma(x+vt), \gamma(t+vx)) = (40\text{ l.y.}, 41\text{ y})$. Not useful.

f) Well, she would have to reset it up by $41 - 9 = \underline{32\text{ y.}}$!

g) This is the wrong question! But answers are: Before, $5.4 + 21 = \underline{26.4\text{ yr}}$; after, $26.4 + 19.2 = \underline{45.6\text{ yr.}}$

h) See above.