April 4 problem set.

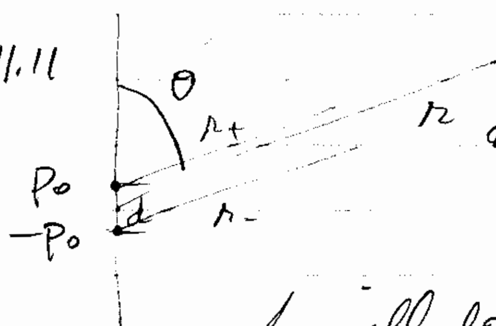
11.3 $$R_{rad} = \underbrace{\frac{\mu_0 I_0^2 d^2}{\omega^2}\frac{\omega^4}{12\pi c}}_{\text{Power } E1}\ \frac{1}{\underbrace{(I_0^2/2)}_{\langle I^2\rangle}}$$

$$= 20\,\Omega\,(kd)^2 = 790\,\Omega\,(d/\lambda)^2$$

11.5 $$R_{rad} = \underbrace{\frac{\mu_0 \overbrace{(I_0 \pi b^2)^2}^{M_0^2} \omega^4}{12\pi c^3}}_{\text{Power } M1}\ \frac{1}{\underbrace{(I_0^2/2)}_{\langle I^2\rangle}}$$

$$= (198\,\Omega)(kb)^4 = 3.08(5)\,(b/\lambda)^4$$

11.11



a) $$A_{dipole} = -\frac{\mu_0}{4\pi} p_0(i\omega)\frac{e^{i(kr-\omega t)}}{r}\hat{z}$$
$$V_{dipole} = \frac{p\cos\theta}{4\pi\varepsilon_0}\left(\frac{1}{r}^{\to 0} - ik\right)e^{i(kr-\omega t)}$$

I will do only the radiation field part

Using $r_\pm = r(1 \mp \frac{d}{2}\cos\theta)$ and keeping only highest order in r, $(1/r_+ \approx 1/r_-)$ (i.e., $1/(r \pm \frac{d}{2}\cos\theta) \approx 1/r$)

$$\vec{A} = \vec{A}^+ + \vec{A}^- = -\frac{\mu_0}{4\pi} p_0(i\omega)\hat{z}\frac{e^{i(kr-\omega t)}}{r}\underbrace{\left(e^{-i\frac{\omega}{c}\frac{d}{2}\cos\theta} - e^{+i\frac{\omega}{c}\frac{d}{2}\cos\theta}\right)}_{(ikd\cos\theta)}$$

$$\vec{A} = -\frac{\mu_0}{4\pi} p_0 \omega k d\cos\theta\,\frac{e^{i(kr-\omega t)}}{r}\hat{z}$$

$$V = \frac{p_0\cos\theta(-ik)(-ikd\cos\theta)e^{i\ldots}}{4\pi\varepsilon_0} = \frac{-p_0}{4\pi\varepsilon_0}k^2 d\cos^2\theta\,\frac{e^{i(kr-\omega t)}}{r}$$

b) Clearly, introducing two oppositely directed terms displaced by d multiplies radiation term by $(-ikd\cos\theta)$. Can also apply this to $\vec{E}$ & $\vec{B}$, or can grind it out from $\vec{A}$ & V. I will do former:

$$\vec{E}_{E1} = -\frac{\mu_0 p_0 \omega^2}{4\pi} \sin\theta \frac{e^{i(kr-\omega t)}}{r} \hat{\theta}, \text{ so}$$

$$\vec{E}_{E2} = -\underbrace{\frac{\mu_0 p_0 \omega^2}{4\pi}(-ikd\cos\theta) \sin\theta}\, \frac{e^{i(kr-\omega t)}}{r} \hat{\theta}$$

and similarly

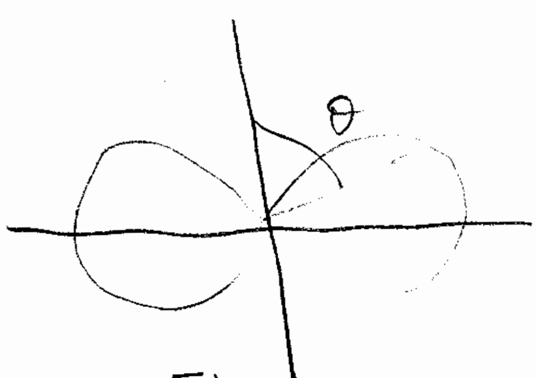
$$\vec{E}_{B2} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}(-ikd\cos\theta)(\sin\theta)\frac{e^{i(kr-\omega t)}}{r} \hat{\phi}$$

c) The Poynting vector for E1 is

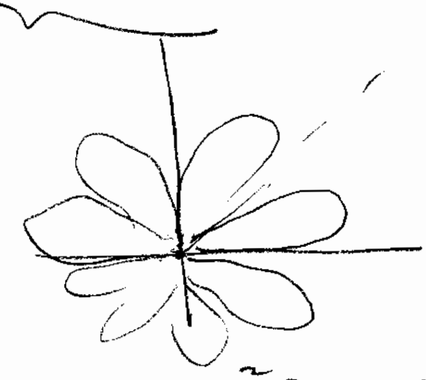
$$S_{E1} = \left(\frac{\mu_0 p_0 \omega^2}{4\pi c}\right)^2 \frac{c}{\mu_0} \frac{\sin^2\theta}{r} \cos^2(kr-\omega t),$$

so

$$S_{E2} = \left(\frac{\mu_0 p_0 \omega^2}{4\pi c}\right)^2 \frac{c}{\mu_0} \underbrace{(kd\cos\theta)^2 \frac{\sin^2\theta}{r}}\cos^2(kr-\omega t)$$



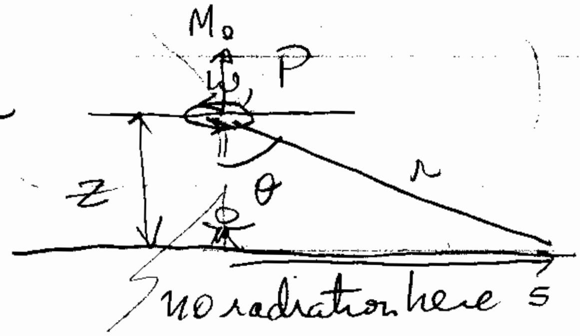
E1



$(\sin\theta\cos\theta)^2 \sim (\cos(2\theta))^2$

E2

11.22



Radiation only:

a) $\langle S \rangle_{m1} = (\quad)\dfrac{\sin^2\theta}{r^2}$

$\dfrac{\mu_0 M_0^2 \omega^4}{32\pi^2 c^3}$

$\sin\theta = s/r$

$\langle S \rangle = (\quad)\left(\dfrac{s^2}{r^4}\right) = (\quad)\dfrac{s^2}{(z^2+s^2)^2}$

$r^2 = z^2 + s^2$

(at FM ~ 100MHz, $\lambda \approx 3m$, assume $z \gg \lambda$)

b) Where does $s^2/(z^2+s^2)^2$ maximize?

$\dfrac{\partial}{\partial s}\left(s^2(z^2+s^2)^{-2}\right) = \cancel{2}s(z^2+s^2)^{-2} + s^2(-2)(\cancel{2}s)(z^2+s^2)^{-3} = 0$

$1 = 2s^2/(z^2+s^2) = 2\left(\dfrac{1}{(z/s)^2+1}\right)$

$(z/s)^2+1 = 2, \quad (z/s)^2 = 1, \quad s = z$

$\boxed{s = z}$ (My hishes z)

$\boxed{s = h}$ where $s^2/(s^2+z^2)^2 = \dfrac{z^2}{4}$

so $\langle S \rangle = (\quad)\dfrac{1}{4z^2}$

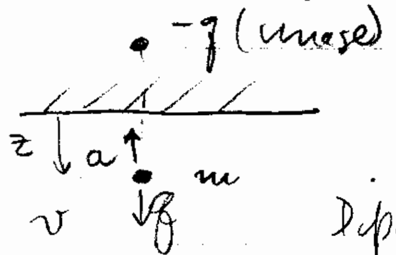
c) $P_{m1} = \dfrac{\mu_0 M_0^2 \omega^4}{12\pi c^3}$ so $\langle S \rangle = \dfrac{12P}{32\pi}\,\dfrac{1}{4}\,\dfrac{1}{z^2}$ at $s = z$

at $z = 200m$, $\langle S \rangle = \dfrac{3}{32\pi}\dfrac{P}{z^2}$

$= .026\ W/m^2 = 2.6\ \mu W/cm^2$

In compliance.

11.25



$$a = \frac{q^2}{4\pi\varepsilon_0 (2z)^2}\frac{1}{m} = \ddot{z}$$

Dipole $p = (2zq)$, $\ddot{p} = 2\ddot{z}q$

$$P = \frac{\mu_0 \ddot{p}^2}{6\pi c} = \frac{\mu_0}{6\pi c}\left(2q\,\frac{q^2}{4\pi\varepsilon_0 (2z)^2 m}\right)^2$$

$$\boxed{P = \frac{(\mu_0 c q^2/4\pi)^3}{6m^2 z^4}}.$$