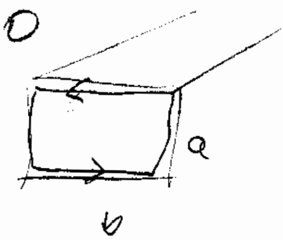
PS. 5

9.26(a). See p (45) lecture notes.

9.27. For TE_{00}, $B_z^0 = B_0 \cos(k_x x)\cos(k_y y) = \text{const.}$

$k_x = k_y = 0$

So, around edge of guide,



$$\oint \vec{E} \cdot d\vec{l} = i\omega B_z^0 (ab)$$

since $B_z = B_z^0 e^{i(kz-\omega t)}$

and $E_{tang} = 0$, so $i\omega B_z^0 (ab) = 0$, or $\underline{B_z^0 = 0}$

on boundary, so it is a TEM mode which is not allowed.

9.28 a) $k_x = n\pi/a = n\,(137)\,m^{-1}$

$k_y = m\pi/b = m\,(311)\,m^{-1}$

$k = \omega/c = 356\ m^{-1}$

Need $k_{tr}^2 = k_x^2 + k_y^2 < k^2$

E.g.,

n	m	k_t	
0	1	311	✓
1	0	137	✓
2	0	274	✓
3	0	411	x
0	2	622	x
1	1	339	✓

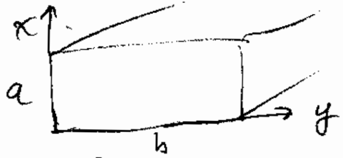
b) To excite lowest mode, TE_{10}, $k_t = 137\ m^{-1}$

$\omega_{min}^{10} = 137\,c = 4.11$ GHz ; to reach next one

$\omega_{20} = 274\,c = 8.22$ GHz; must keep

$4.11 < \omega < 8.22$ GHz

$.65 < \nu < 1.3$ GHz



$B_z^0 = 0$

9.30. as in class separation of variables,

$$E_0^z = (A\sin(k_x x) + B\cos(k_x x)) \times (C\sin(k_y y) + D\cos(k_y y))$$

B.C. are now that $E^{\parallel} = 0$, or

at $y = 0, b$ $\quad E_x = 0 = \frac{i}{k_t^2} k_z \left(\frac{\partial E_z}{\partial x}\right)$, $D = 0$, $k_y b = n\pi$

at $x = 0, a$ $\quad E_y = 0 = \frac{i}{k_t^2} k_z \left(\frac{\partial E_z}{\partial y}\right)$, $B = 0$, $k_x a = m\pi$

a) So $\boxed{E_0^z = A' \sin(k_x x)\sin(k_y y)}$

There is no 01 or 10 mode: Lowest is 1,1.

Cut off frequencies are at

$$k^2 = k_x^2 + k_y^2 = \pi^2\left(\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2\right) = \omega^2/c^2$$

b) or $\boxed{\omega = \pi c\left(\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2\right)^{1/2}}$

c) $$\frac{TM_{1,0}}{TE_{1,1}} = \frac{\pi c\left(\frac{1}{b}\right)}{\pi c\left[\left(\frac{1}{b}\right)^2 + \left(\frac{1}{a}\right)^2\right]^{1/2}} \quad (\text{if } b > a)$$

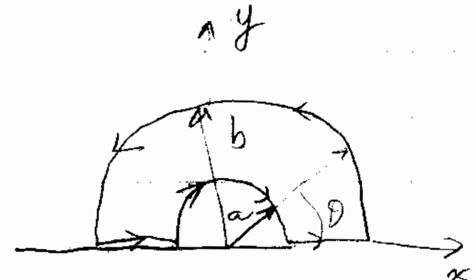
$$= \left(1 + (b/a)^2\right)^{1/2}$$

d) Wave vel.: $v_g = c\sqrt{1 - (\omega_{nm}/\omega)^2}$

$v_{ph} = c\,/\sqrt{1 - (\omega_{mn}/\omega)^2}$

Problems:

10.10.



$I = k(t)$

Need $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(t' = t - R/c)\, d\tau'}{R} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(t' = t - R/c)\, dl}{R}$

where $\vec{R} = \vec{r} - \vec{r}'$. Here $\vec{r} = 0$.

$$A_x = \frac{\mu_0}{4\pi}\left[-\int\left(\frac{I(t')\sin\theta\, b}{b} - \frac{I(t')\sin\theta\, a}{a}\right) d\theta \right.$$ ← around circle

$$\left. + 2\int_a^b \frac{I(t')\;dx}{x}\right]$$ ← along straight section

The A_y and A_z components are either zero or cancel.

$$A_x = \frac{\mu_0}{4\pi}\left[-\int_0^\pi \left(k(\underbrace{t - b/c}_{t' = t - b/c})\sin\theta - k(\underbrace{t - a/c}_{t' = t - a/c})\sin\theta\right)d\theta\right.$$

$$\left. + 2\int_a^b \frac{k(\overbrace{t - x/c}^{t' = t - x/c})}{x}\, dx\right]$$

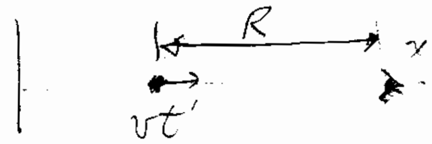
$$= \frac{\mu_0 k}{4\pi}\left[\left(+\frac{b}{c} - \frac{a}{c}\right)\underbrace{\int_0^\pi \sin\theta\, d\theta}_{2}\right.$$

$$\left. + 2t \ln x\Big|_a^b - 2\left(\frac{b-a}{c}\right)\right]$$

$$= \frac{\mu_0}{4\pi}k\;\; 2t\ln(b/a)$$

$A_x = \frac{\mu_0}{2\pi}\ln(b/a)\, t$, $\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -\hat{x}\,\frac{\mu_0}{2\pi}\ln(b/a)$

$d\vec{B}/dt \to \vec{E}$; Cannot get $\vec{\nabla}\times\vec{A}$ because to take curl requires knowing $\vec{A}$ as f'n. of position.



lgh 10.18 $E_x = \frac{q}{4\pi\epsilon_0} \frac{\gamma(x-vt)}{R'^3}$ where $R' = \sqrt{\gamma^2(x-vt)^2+y^2+z^2}$
$= \gamma(x-vt)$

In terms of $R = x - v(t - R/c)$

$$R = \frac{x-vt}{1-v/c}$$

So $E_x = \frac{q}{4\pi\epsilon_0} \frac{\gamma(x-vt)}{(\gamma(x-vt))^3} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\gamma(x-vt)}\right)^2.$

If you insist on writing it in terms of R,

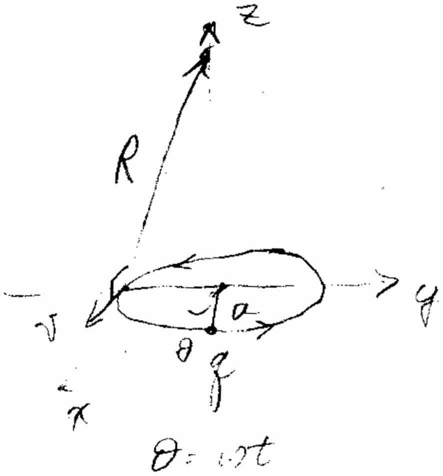
$$E_x = \frac{q}{4\pi\epsilon_0} \frac{1}{R^2} \frac{1}{\gamma^2(x-vt)^2} \cdot \underbrace{\frac{(x-vt)^2}{(1-v/c)^2} \quad (1-v^2/c^2)}_{\frac{\cancel{(1-v/c)}(1+v/c)}{\cancel{(1-v/c)}(1-v/c)}}$$

$$E_x = \frac{q}{4\pi\epsilon_0} \frac{1}{R^2} \frac{1+v/c}{1-v/c}$$

$$\boxed{E_x = \frac{q}{4\pi\epsilon_0} \frac{1}{R^2} \left(\frac{c+v}{c-v}\right)}$$

Fields to left are $c-v/c+v$, in $-E_x$ dir.
$B_x = 0$ in general.

10.13



Need $V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R - \vec{R}\cdot\vec{v}/c}\right]_{ret}$

Fortunately, R is constant $= \sqrt{z^2+a^2}$

$\vec{v} = \hat{x} a\omega \sin\omega t' + \hat{y} a\omega \cos(\omega t')$

and $t' = t - R/c$ so

$\vec{v} = -a\omega(\hat{x}(-)\sin(\omega t - \omega R/c) + \hat{y}\cos(\omega t - \omega R/c))$

and $\vec{R} = \hat{x} a \cos\omega t' + \hat{y} \sin\omega t'$

$= a(\hat{x} \cos(\omega t - \omega R/c) + \hat{y}\sin(\omega t - \omega R/c))$

So $\vec{R}\cdot\vec{v} = 0$ (by inspection)

$\vec{R} = \hat{z} z + \hat{x} a \sin(\omega t - R/c) + \hat{y} a \cos(\omega t - R/c)$

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{z^2+a^2}} \qquad \vec{A} = \frac{\vec{v}}{c^2}V = \frac{-a\omega(\hat{x}(-)\sin(\omega t - \omega R/c) + \hat{y}\cos(\omega t - \omega R/c))}{c^2} \times \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{z^2+a^2}}}$$

10.20. $\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} = \frac{q}{4\pi\epsilon_0}\left\{\frac{-z}{(z^2+a^2)^3}\hat{z}\right.$

$+ a\omega^2(\hat{x}\cos(\omega t - \omega R/c) + \hat{y}\sin(\omega t - \omega R/c))$

$\left.\times \frac{1}{c^2\sqrt{z^2+a^2}}\right\}$

$$\boxed{\vec{B} = \frac{1}{c}\hat{R}\times\vec{E}}$$

; Since the position of the charge is $\hat{x}\cos(\omega t - \omega R/c) + \hat{y}\sin(\omega t - R/c)$, the $\hat{R}$ and $\vec{E}_{xy}$ are $\parallel$ and do not contribute,

$\hat{R} = \frac{z\hat{z} + \hat{x} a \cos(\sim) + \hat{y} a \sin(\sim)}{\sqrt{z^2+a^2}}$

$$B_z = \frac{q}{4\pi\epsilon_0}\frac{z a^2\omega^2}{\left(\sqrt{z^2+a^2}\right)^2}\left(\cos^2(u)+\sin^2(u)\right)$$

$$= \frac{q}{4\pi\epsilon_0}\frac{a^2\omega^2}{(z^2+a^2)} \quad e^{ir}.$$

Cannot get it from $\vec{\nabla}\times\vec{A}$.