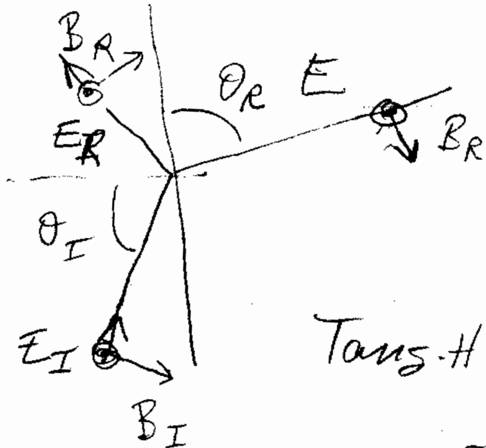
-P16

9.16. Do "S" Polarization case. E out of page



Same eq'ns as in "P" case, except $E \rightleftarrows B$, $B = E/v$

Tang. H $\quad \frac{1}{\mu_1}(B_I \cos\theta_I - B_R \cos\theta_I) = \frac{1}{\mu_2} B_T \cos\theta_T$

Tang. E $\quad E_I + E_R = E_T$

Tang H becomes $\quad \cancel{\frac{1}{\mu_1 v_1}} (E_I - E_R) = \underbrace{\frac{\mu_1 v_1}{\mu_2 v_2}}_{\beta} E_T \underbrace{\frac{\cos\theta_T}{\cos\theta_I}}_{\alpha}$

$$E_T = E_I \left(\frac{2}{1+\beta\alpha}\right)$$

$$E_R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right) E_I$$

Problem: Build a spread sheet to calculate, for $n_1 = 1$, $n_2 = 1.5$, $\mu_1 = \mu_2$, E_R/E_I and E_T/E_I versus θ_I.

Trans & Refl. coeff:

$$R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$$

$$T = \underbrace{\frac{\varepsilon_2}{\varepsilon_1} \frac{v_2}{v_1}}_{\beta} \underbrace{\left(\frac{\cos\theta_T}{\cos\theta_I}\right)}_{\alpha} \left(\frac{2}{1+\alpha\beta}\right)^2 = \alpha\beta\left(\frac{2}{1+\alpha\beta}\right)^2$$

$$R + T = 1$$

~~P16~~ P17

Problems 9.16, 9.18, 9.19, 9.20, +

a) Fresnel eq'ns S & P

b) Brewster ∡ .6 to 2.5 μ.

9.18. a) Glass: take $\rho \sim 10^{12}$ (middle of range), p 286.

$\varepsilon \sim 10$ (could be in this range) $\times \varepsilon_0$

So $\tau = \epsilon/\sigma = 8.8(-11) \times 10^{12} = \underline{88\,sec}$.

(Pretty good glass!)

b) $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$, $\mu = 4\pi(-7)$; $\sigma = [1.59(-8)]^{-1}$;

$\omega = 2\pi \times 10^{10}$; $\delta = \underline{.63\,\mu}$.

Make coating $\sim \underline{1-2\,\mu}$.

c) In air, at $\underline{1 MHz}$, $\omega = 6.28(6)/s$, $v = 3(8)m/s$, $k = 2.1(-2)m$

$\lambda = 3.00$ cm

In Cu, $4\pi(-7)$, $\sigma = [1.68(-8)]^{-1}$, $\varepsilon \sim 8.8(-12)$

Is it a good conductor? $\omega\varepsilon/\sigma \sim 10^{-12} \Rightarrow$ yes!

So $k_{cu} = \sqrt{\mu\sigma\omega/2}\, e^{i\pi/4} \simeq 2.16/\sqrt{2} \times 10^4\, m^{-1}\, e^{i\pi/4}$

$\lambda \simeq 410\,\mu$, $v = \omega/k_{Real} = \underline{411\,m/s}$.

9.19. a) Skin depth $E = E_0 e^{-i(\sim)} e^{-x/\delta}$

$k \doteq \sqrt{\mu\varepsilon\omega}\,(1 + i\,\sigma/2\varepsilon\omega)$ so

$1/\delta = \mathrm{Im}\,k = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}}$, $\delta = \frac{2}{\sigma}\sqrt{\varepsilon/\mu}$

For $\varepsilon = 80.1\,\varepsilon_0$ (water)

$\mu \simeq 4\pi(-7)$ (")

$\sigma \simeq [2.5(5)]^{-1}$

$\delta = \underline{1.1(4)\,m}$

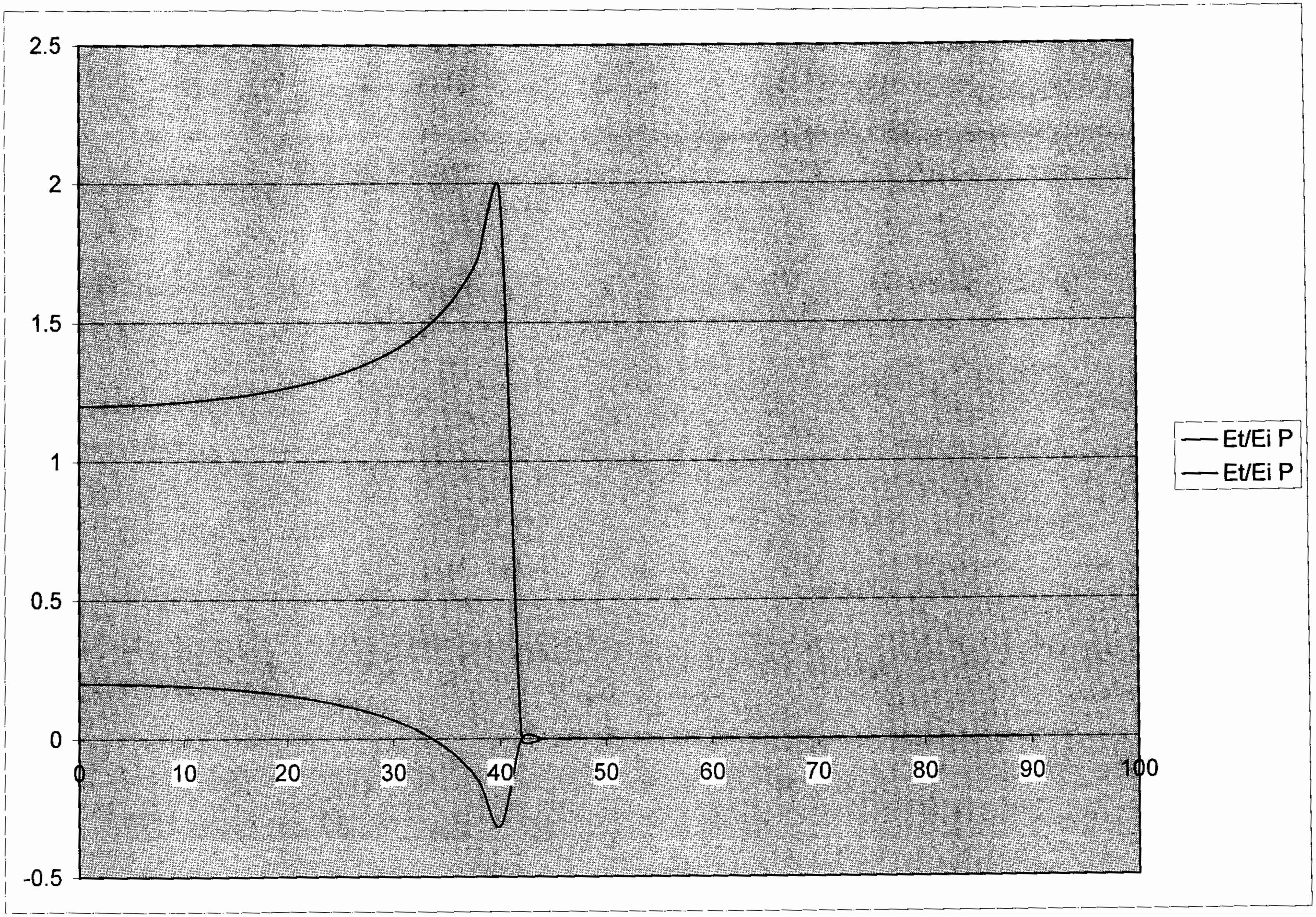
b) In good conductor $k = \frac{1}{\delta} + \frac{i}{\delta}$ where $\frac{1}{\delta} = \frac{2\pi}{\lambda}$

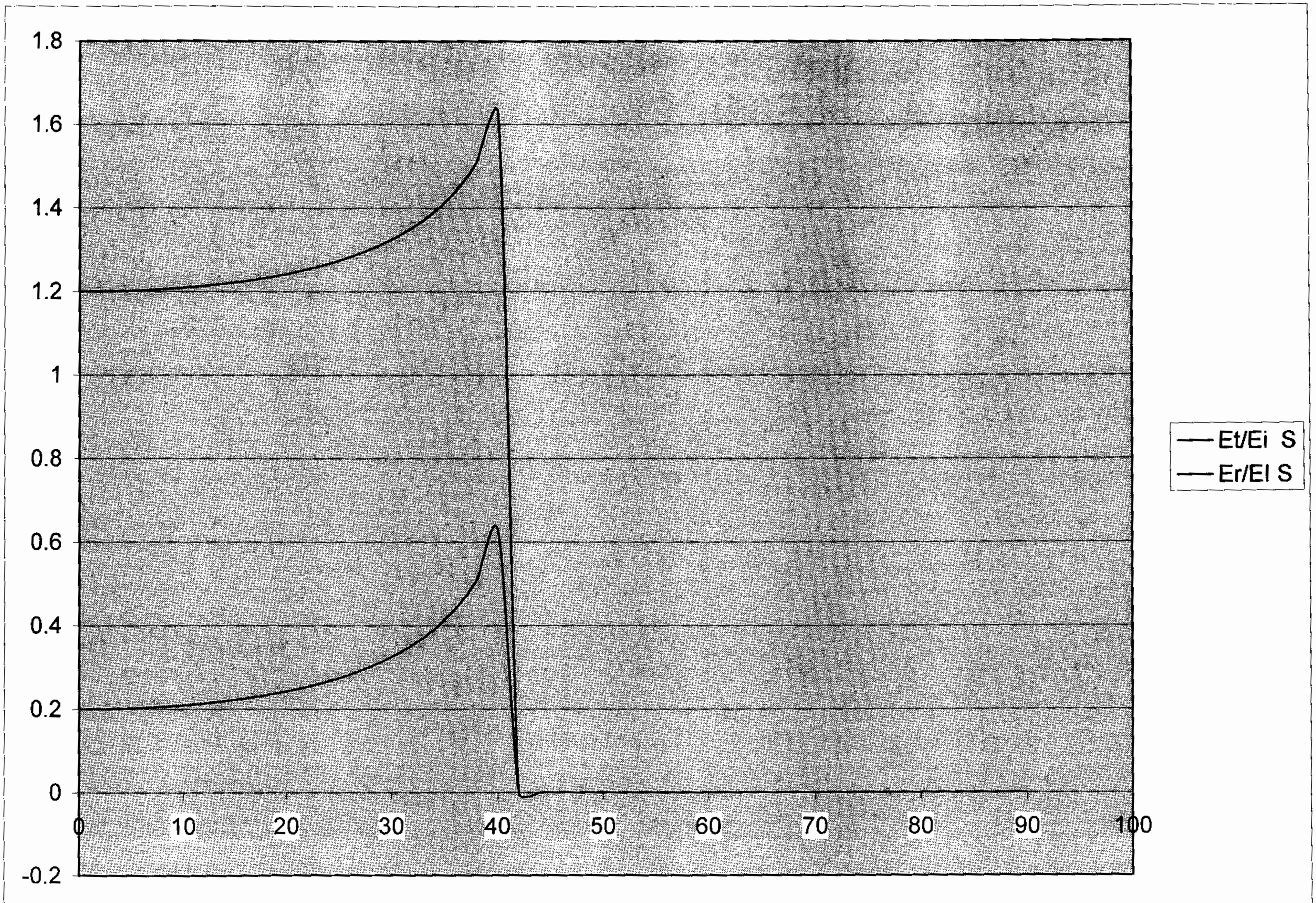
so $\delta = 2\pi/\lambda$. $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$, $\mu = 4\pi(-7)$, $\sigma = 10^7$,

$\omega = 10(15) \Rightarrow \delta = \underline{12.6\,nm}$. $<< \lambda$ air of visible light.

c) Done in class.

P18-21

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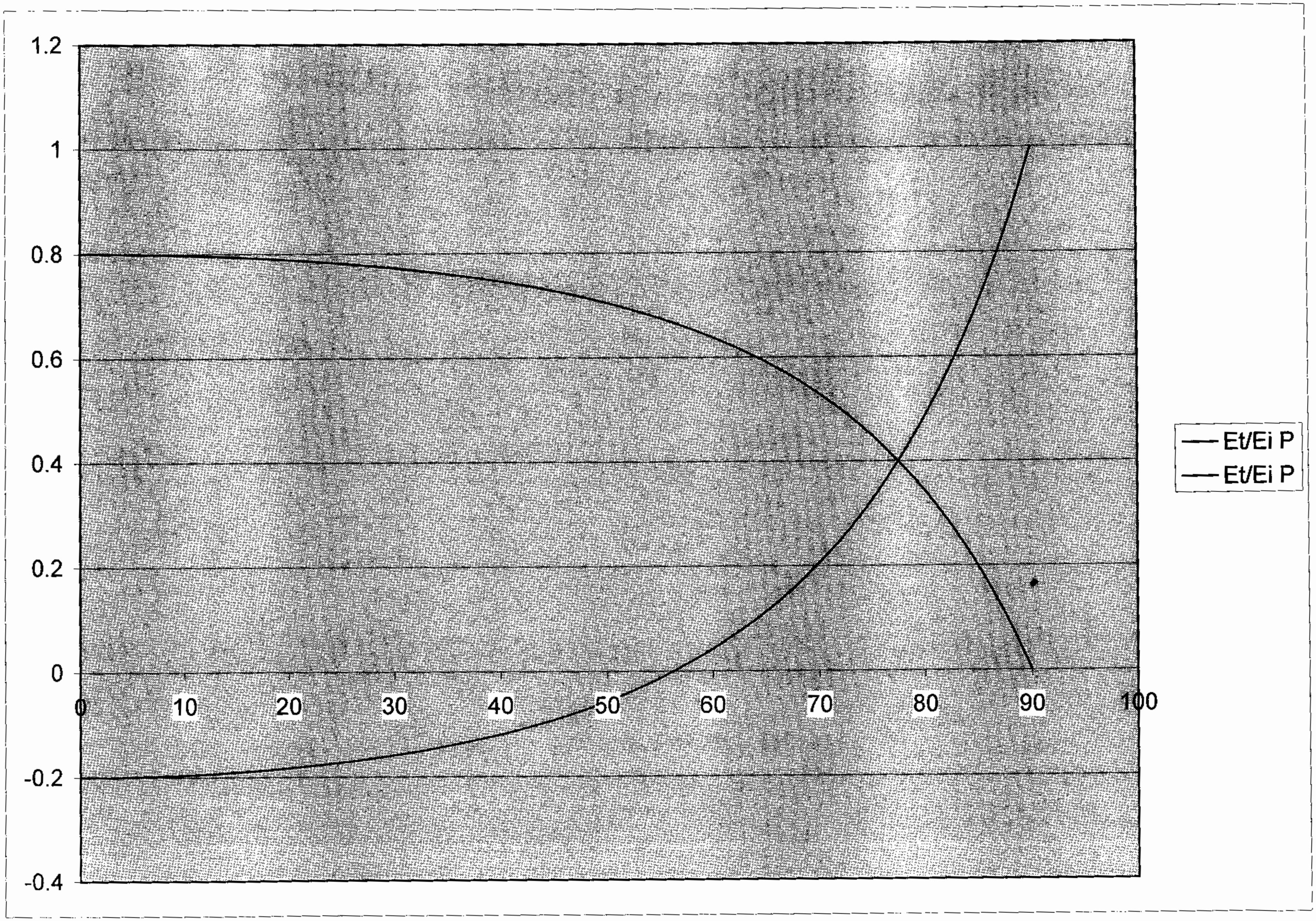
Et/Ei S
Er/EI S


1.2
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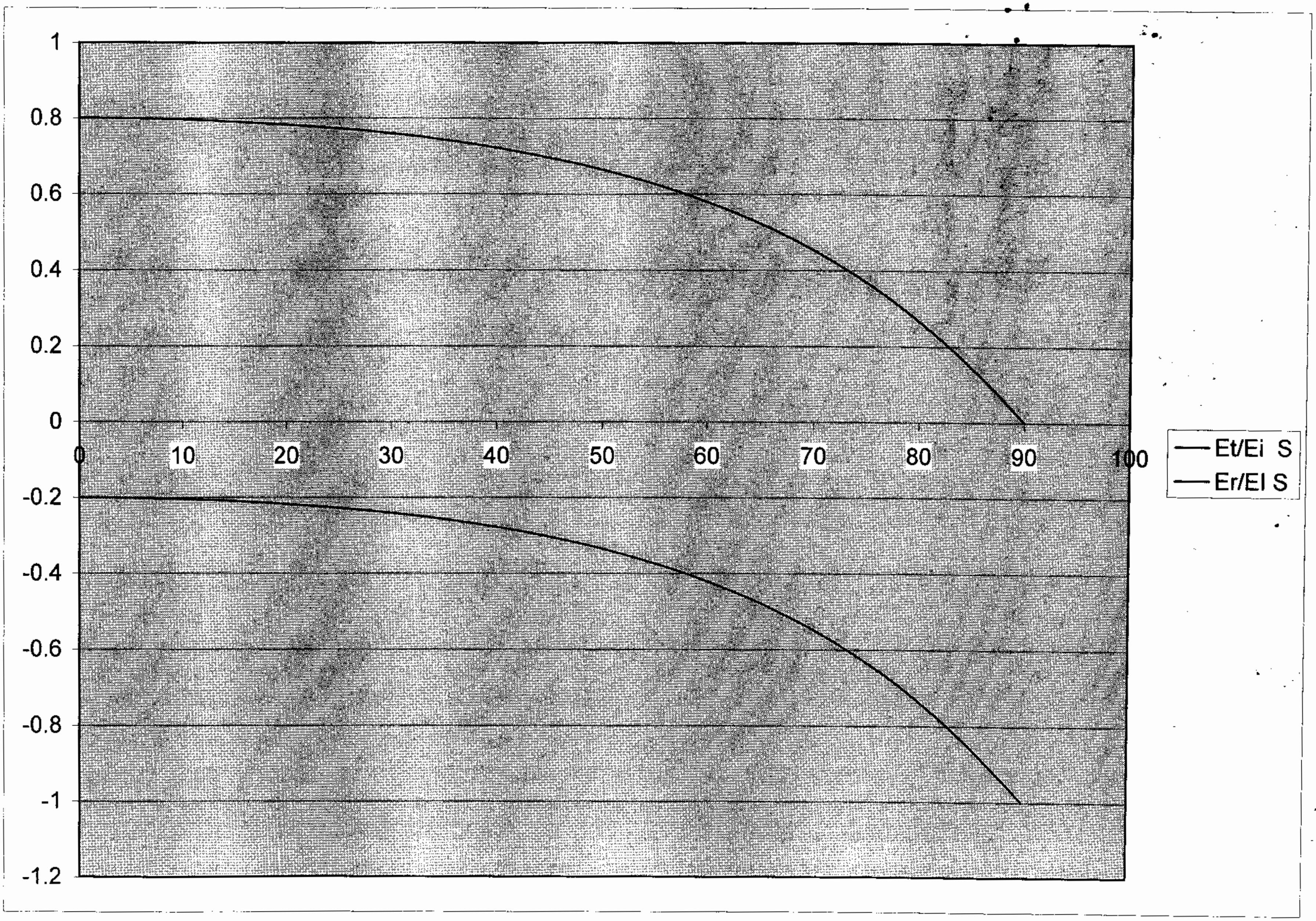
Et/Ei P



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Er/EI S

Problems due Feb. 26 (they may be put in my mailbox on Tuesday) : Problem 9.21,9.23 (a),9.25 ; plus

1) A conducting material can be heated by induction heating by exposing it to a time varying magnetic field. Suppose a good conductor (with skin depth delta) is exposed to a magnetic field which has a component tangential to the surface given by $B=B_o \exp(-iwt)$. The surface lies in the xy plane, and B is in the y direction.

a) Using the boundary conditions on B, find B(z,t) inside the material.

b) This B will cause an electromagnetic wave to propagate into the material. Using the form of this wave developed in class, calculate the Poynting vector into the surface, evaluated just at the surface.

c) Using the value of E for this wave, and j=sigma*E, find the surface current flowing in the conductor. By integrating over z, find the current per unit length flowing in the surface of the conductor (this extends roughly over the skin depth of the conductor). Make a clear sketch to show that you understand the geometry and which way the surface current flows.

d) Using the boundary condition on tangential B which applies if there is a surface current (extending over a thickness of several skin depths), show that the current found in (c) is just enough to kill the incident B.

e) Calculate the ohmic heating in the surface by integrating j dot E over the volume of the material, and show that it agrees with the answer to (b).

f) Calculate the numerical value of the skin depth for Cu at 60 Hz. How large a piece of Cu would you expect to be able to heat this way?

2) An electromagnetic wave at 1 kHz is sent OUT of sea water (conductivity 5/ohm-m) in normal incidence into air.

a) Show that sea water is a good conductor at this frequency and find the skin depth.

b) Write the form of the incident, reflected and transmitted waves. Assume the wave is sent in the positive z direction and is polarized in the x direction. Describe the nature of each wave: damped, traveling, etc.

c) Find the ratio of the strength of the electric field of the incident wave just below the surface to that of the transmitted wave just above the surface, to first order in the skin depth.

d) What fraction of the power reaching the surface is transmitted through the surface?

Pset 4 :

9.21. Assume good conductor,

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} = 2.57(-9)\,m \ ; \ k = \omega/c = 1.33(7)\,m^{-1}$$

$\mu = 4\pi(-7)$; $\sigma = 6(7)$; $\omega = 4(15)$

Reflection coeff $= 1 - 2k\delta = \underline{93.1\%}$

9.23 Inside a uniformly charged sphere, using Gauss's Law, $\rho = (q/\tfrac{4}{3}\pi a^3)$ so

$$E(r)4\pi r^2 = q_{encl}/\varepsilon_0 = (r^3/a^3)\,q/\varepsilon_0 \text{ or}$$

$$E(r) = \frac{q}{4\pi\varepsilon_0}\frac{r}{a^3}, \text{ linear with } r.$$

So if displace nucleus relative to center of sphere, restoring force is $E(x)q$ or

$$m\ddot{x} = -\underbrace{\frac{q^2}{4\pi\varepsilon_0}\frac{1}{a^3}}_{\text{"}k\text{"}}x$$

$$\omega = \sqrt{k/m} = \sqrt{\frac{q^2}{4\pi\varepsilon_0 a^3}\frac{1}{m}}$$

$1/4\pi\varepsilon_0 = 9(9)$; $q = 1.6(-19)$; $a = 0.5(-10) = 1\,\text{Å}$; $m = 1.67(-27) = m_p$

$\underline{\omega = 1.04(15)\ \text{ra/s}}$ which is in infrared.

So model is wrong, but in right ballpark.

9.25. $$k = \frac{\omega}{c}n = \frac{\omega}{c}\left(1 + \frac{Nq^2}{2m\varepsilon_0}\frac{1}{\omega_0^2-\omega^2}\right) \quad \text{Let } \omega_p^2 = \frac{Nq^2}{2m\varepsilon_0}$$

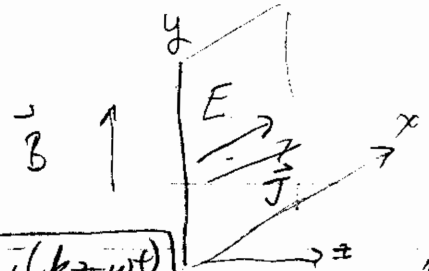
$$\text{so } dk = \frac{d\omega}{c}\left(1+\frac{\omega_p^2}{\omega_0^2-\omega^2}\right) + \frac{\omega}{c}\left(\frac{\omega_p^2}{(\omega_0^2-\omega^2)^2}(+2\omega)\right)d\omega$$

$$= \frac{d\omega}{c}\left(1 + \frac{\omega_p^2}{\omega_0^2-\omega^2} + \frac{2\omega^2\omega_p^2}{(\omega_0^2-\omega^2)^2}\right)$$

$$= \frac{d\omega}{c}\left(1 + \frac{(\omega_p^2\omega_0^2 + \omega_p^2\omega^2)}{(\omega_0^2-\omega^2)^2}\right) = \frac{d\omega}{c}\left(1 + \omega_p^2\frac{\omega_0^2+\omega^2}{(\omega_0^2-\omega^2)^2}\right)$$

$$v_{gr} = \frac{d\omega}{dk} = c\,\frac{1}{\left(1 + \omega_p^2\frac{\omega_0^2+\omega^2}{(\omega_0^2-\omega^2)^2}\right)}$$ always > 1, so $v_{gr} < c$.

(1)

Extra: 

$k = \frac{1}{\delta} + \frac{i}{\delta} = \frac{\sqrt{2}}{\delta} e^{i\pi/4}$

a) $\boxed{\vec B = \hat y B_0 e^{i(kz-\omega t)}}$ since B'' continuous.

b) Need E inside. Since $\vec k \times \vec E = \omega \vec B$,

$$E_0 = \frac{\omega}{k} B_0 , \quad \boxed{E = \hat x \frac{\omega \delta B_0}{\sqrt{2}} e^{i(kz-\omega t)} e^{-i\pi/4}}$$

$$\langle S \rangle = \frac{1}{2\mu_0}(\vec E_0 \times \vec B_0)\cos\phi = \boxed{\frac{1}{2\mu_0} \frac{\omega\delta}{\sqrt{2}} B_0^2 \cos(\pi/4)} \underbrace{e^{-2kz}}_{\substack{=1 \\ \text{at } z=0}}$$

$$\boxed{\langle S \rangle = \frac{\omega \delta B_0^2}{4} \text{ at } z=0}$$

c) $\vec J$ is in x-direction

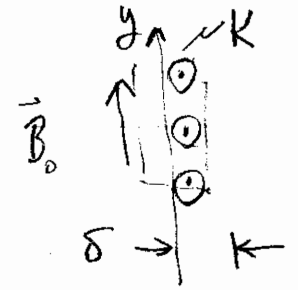
$$\vec J = \hat x \frac{\omega\delta\sigma B_0}{\sqrt{2}} e^{i(kz-\omega t)} e^{-i\pi/4} = \vec E \sigma$$

To get $\vec K$, $\int \vec J \, dz$ (current / dy)

$$\vec K = \hat x \frac{\omega\delta\sigma}{\sqrt{2}} B_0 e^{-i\omega t - i\pi/4} \underbrace{\int_0^\infty e^{ikz} dz}$$

$$\frac{1}{ik} = \frac{\delta}{\sqrt{2}} e^{-i\frac{3}{4}\pi}$$

$$\vec K = \hat x \frac{\omega \delta^2 \sigma}{2} (-1) B_0$$

d) 

Since B_0 is killed by this K, Stokes's Law requires $B_0 = \mu_0 K$ or

$$B_0 = \left(\frac{\omega \delta^2 \sigma}{2}\right) \mu_0 \ ; \quad \delta^2 = \frac{2}{\mu_0 \sigma \omega} \text{ so}$$

$$B_0 = \frac{\cancel{\omega} \cancel{2} \cancel{\sigma}}{\cancel{\mu_0} \cancel{\sigma} \cancel{\omega} \cancel{2}} \cancel{\mu_0} B_0 = B_0 \checkmark$$

Walls.

e) Integrate $\vec{J}\cdot\vec{E}$ over z to get Power/area

$$= \underbrace{\frac{\omega\delta B_0}{\sqrt{2}}}_{E} \underbrace{\frac{\omega\delta B_0}{\sqrt{2}}\sigma}_{J} \left\langle \int \underbrace{\cos^2(z/\delta-\omega t)}_{1/2} \underbrace{e^{-2z/\delta}\,dz}_{\delta/2} \right\rangle$$

$\sigma = \frac{2}{\mu\omega\delta^2}$, so

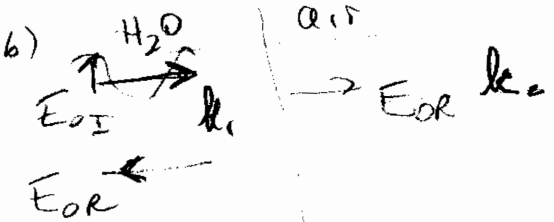
$$= \frac{\omega\cancel{\delta} B_0}{\sqrt{2}} \frac{\cancel{\omega}\cancel{\delta} B_0}{\sqrt{2}} \frac{\cancel{2}}{\mu\cancel{\omega}\cancel{\delta}^2} \frac{\delta}{4} = \frac{\omega\delta}{4\mu} B_0^2 = \langle S \rangle$$

from (b).

2) a) $\tau = \varepsilon/\sigma = \frac{80 \times 8.8(-12)}{5} = 1.4(-10)$ sec,

$\omega\tau = 10^{3} \times 2\pi\tau \sim 10^{-6} << 1$; $\delta = \sqrt{\frac{2}{\sigma\omega\mu}} = 7.1$ m

b)



In water $\vec{E} = \hat{x}\left(E_{0I}\left(e^{i(k_1 z - \omega t)}\right) + E_{0R}\, e^{i(-k_1 z - \omega t)}\right)$, Damped

In air $\vec{E} = \hat{x} E_{0R}\, e^{i(k_2 z - \omega t)}$, traveling

c) $E_{0T}/E_{0I} = \frac{2k_1}{k_1 + k_2}$ $\quad \tilde{k}_1 = \frac{\sqrt{2}}{\delta} e^{i\pi/4} \sim .2\ m^{-1}$

so $k_1 >> k_2$ $\quad k_2 = \omega/c = 2 \times 10^{-5}\ m^{-1}$

E_{0T} (above)/E_{0I} (inc.) $\simeq 2$.

d) $R = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2 \simeq 1 - 2k_2\delta = 1 - 2.8(-4)$

$\boxed{T = 1 - R = 2.8 \times 10^{-4}}$