③ P-2

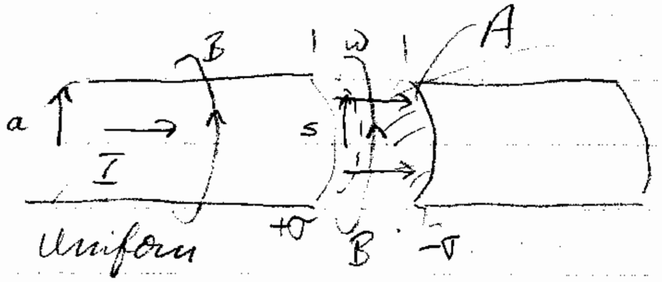
If $\vec{J}, \rho$ are fixed in space, $\vec{B}$ + $\vec{E}$ are also OK.
But if E + B are t-dep., each generates other:

$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\, \partial\vec{E}/\partial t \qquad \vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$

Rate of change of each causes other to circulate about it! So all of old Ampere's law pictures work.

Probs:

7.31



$|\vec{E}| = \sigma/\varepsilon_0 = Q/\varepsilon_0 A$

$I_d = \varepsilon_0 \frac{d\vec{E}}{dt}\cdot\vec{A} = I$

$$\int\vec{B}\cdot d\vec{l} = \mu_0(I + I_d) = 2\pi s B$$

① In wire, $s > a$, $2\pi s B = \mu_0 I$; $B = \frac{\mu_0 I}{2\pi s}$

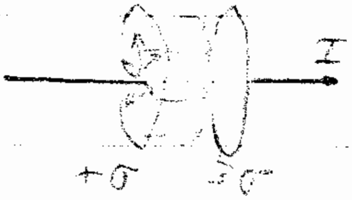
$s < a$, $2\pi s B = \mu_0(\pi s^2/\pi a^2) I$; $B = \frac{\mu_0 I s}{2\pi a^2}$

② In gap, $s > a$ $\quad 2\pi s B = \mu_0 I_d = \mu_0 I$, $B = \frac{\mu_0 I}{2\pi s}$

$s < a$, $\quad 2\pi s B = \mu_0 \underbrace{\frac{\varepsilon_0 dE}{dt}}_{\frac{I}{\pi a^2}} \cdot \pi s^2$, $B = \frac{\mu_0 I s}{2\pi a^2}$

$\vec{J}_{disp}$ exactly takes place of $\vec{J}$ in wire.

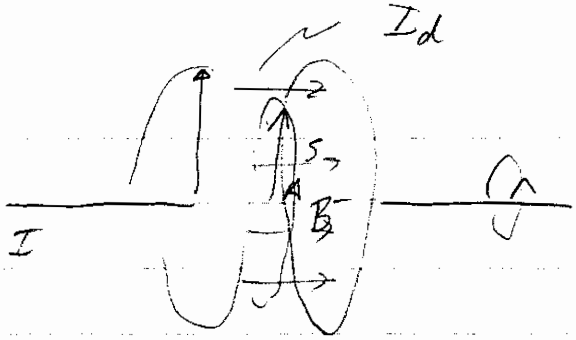
7.32:



Now the gap is different. J_D is more spread out, E still constant in space but spreads out. Find B in gap.

~~②~~ P4

7.32



a)

① In gap

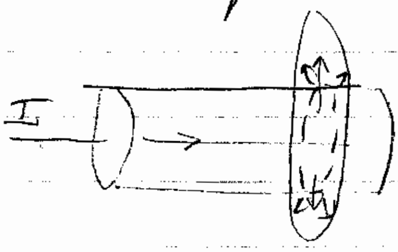
As in 7.31, $I_d = I$, so, $s < a$

$$2\pi s B = \mu_0 I\left(\frac{\pi s^2}{\pi a^2}\right), \quad B = \frac{\mu_0 I}{2\pi}\frac{s}{a^2}$$

$$B = \frac{\mu_0 I}{2\pi s} \qquad s > a$$

b) ② around wire $B = \frac{\mu_0 I}{2\pi s}$.

c) Complete circuit with o-juice can:



$$\oint B\cdot dl = \mu_0 \underbrace{\left(\frac{\pi s^2}{\pi a^2}\right)\varepsilon_0 I}_{I_d \text{ through open end.}}$$

If use can instead, set

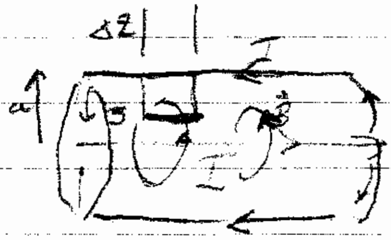
$$\mu_0 \varepsilon_0 \; I_{net} = \mu_0\varepsilon_0\left(\underset{\substack{\uparrow\\ \text{end}}}{I} - \underset{\substack{\uparrow\\ \text{out edge of can}}}{\left(\frac{\pi a^2 - \pi s^2}{\pi a^2}\right)}I\right)$$

$$= \mu_0\varepsilon_0 \; I\left(\frac{\pi s^2}{\pi a^2}\right) = \mu_0\varepsilon_0 I\frac{s^2}{a^2}$$

Same as
I_d through end of can.

Po

7.33 : (7.6) :



a) $I = I_0 \cos(\omega t)$, giving rise to
$2\pi s\, B_\phi = \mu_0 I_0 \cos(\omega t)$ or

$$\phi = \int_s^a \frac{\mu_0 I}{2\pi s}\,\Delta z\, ds = \frac{\mu_0 I}{2\pi}\,\Delta z \ln(a/s)$$

Since ϕ depends on time, $\int \vec{E}\cdot d\vec{l} = -\dot{\Phi}$

$$E\,\cancel{\Delta z} = -\frac{\mu_0 \dot{I}}{2\pi}\,\cancel{\Delta z} \ln(a/s)$$

$$\vec{E} = -\frac{\mu_0 \omega I_0 \sin(\omega t)}{2\pi} \ln(a/s)\,\hat{z}, \text{ or}$$

$$J_d = -\frac{\mu_0 \varepsilon_0 \omega^2 I_0 \cos(\omega t)}{2\pi} \ln(a/s) \quad \hat{z}$$

b) $$I_d = -\frac{\mu_0 \varepsilon_0 \omega^2 I_0 \cos(\omega t)}{2\pi} \int_0^a \ln(a/s)\, 2\pi s\, ds$$

$$I_d = -\mu_0 \varepsilon_0 \omega^2 I_0 \cos(\omega t)\, \cancel{2\pi} \underbrace{\int_0^a [(\ln a) s\,ds - (\ln s) s\,ds]}$$

$$\underbrace{\ln a \left(a^2/2\right) - \left(\frac{a^2}{2}\ln a - \frac{a^2}{4}\right)}_{a^2/4}$$

$$I_d = -\left(\mu_0 \varepsilon_0 \omega^2 \frac{\cancel{2}\pi a^2}{4}\right) I$$

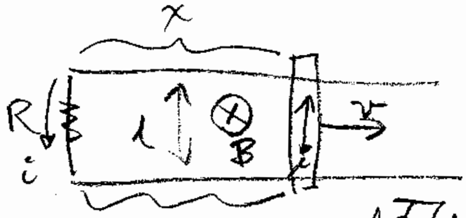
c) Set $\mu_0 \varepsilon_0 \omega^2 \frac{\pi a^2}{4} = .01$

$\mu_0 \varepsilon_0 = \frac{1}{c^2}$, so $\left(\frac{\omega}{c}\right)^2 \frac{\pi a^2}{4} = .01$, $\omega = \sqrt{\frac{4(.01)}{\pi}}\, \frac{c}{a}$

$$\boxed{\omega = 3.3(10)\ \text{ra/sec}}$$

Problems:

7.7



a) $i = \frac{EMF}{R} = -\frac{d\Phi/dt}{R} = -\frac{\frac{d}{dt}\left[Bxl\right]}{R} = -\frac{vBl}{R}$

Current fights increase of flux, ccw by right handrule.

b) Force $= \int i\,dl \times B = ilB = \frac{vB^2l^2}{R}$;

Force opposes motion, to left.

c) F = ma ⇒ $v\frac{B^2l^2}{R} = m\frac{dv}{dt}$

$$\frac{B^2l^2}{Rm}\underbrace{\int_0^t dt}_{t} = \underbrace{\int_{v_0}^{v}\frac{dv}{v}}_{\ln v/v_0}$$

$$v = v_0 e^{-t/\tau} \text{ where } \tau = \frac{Rm}{B^2l^2}$$

d) Energy lost in time t $= \int i^2 R\,dt$

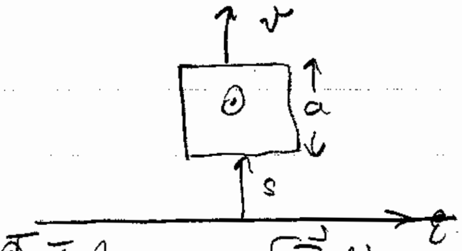
$$= \int \frac{v^2B^2l^2}{R^2}R\,dt$$

$$= \frac{v_0^2B^2l^2}{R}\frac{\tau}{2}\underbrace{\int_0^t \left(e^{-t/\tau}\right)^2 \frac{dt}{\tau}}_{\left(1-e^{-2t/\tau}\right)} \overset{?}{=} \frac{m}{2}v_0^2\left(1-e^{-2t/\tau}\right)$$

$$\frac{\cancel{v_0^2B^2l^2}}{\cancel{R}}\frac{\cancel{Rm}}{\cancel{B^2l^2}}\frac{1}{\cancel{2}} \overset{?}{=} \frac{\cancel{m}}{\cancel{2}}\cancel{v_0^2}$$

$1 = 1$ ✓.

7.8



a) Φ = Flux = $\int \vec{B}\cdot d\vec{a}$; $B_{\text{long wire}} = \frac{\mu_0 I}{2\pi S}$

$$\Phi = \int_s^{s+a} \underbrace{\left(\frac{\mu_0 I}{2\pi S}\right)}_{\text{"}B\text{"}} \underbrace{(a\,ds)}_{\text{"}da\text{"}} = \frac{\mu_0 I}{2\pi} \ln\left(s+a/s\right).$$

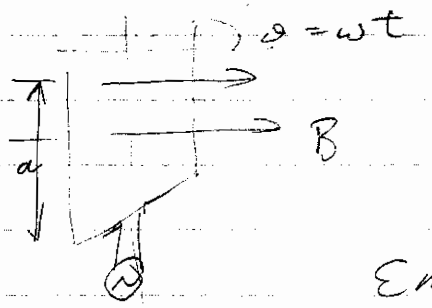
b) emf = $-d\Phi/dt$, take time derivative

$$= \frac{\mu_0 I}{2\pi} \frac{d}{dt}\left(\ln\left(\frac{s+a}{s}\right)\right)$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{s+a} - \frac{1}{s}\right] \underbrace{\frac{ds}{dt}}_{= v} = \frac{\mu_0 I v}{2\pi}\left[\frac{1}{s+a} - \frac{1}{s}\right]$$

c) If Moves to right $d\Phi/dt = 0 =$ emf.
Direction of emf opposes decreasing flux
out of page $\Rightarrow$

7.10.

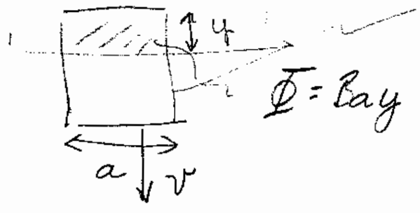


Φ thru coil = $Ba^2 \sin\omega t$

EMF = $-d\Phi/dt = -\omega B a^2 \cos\omega t$

P3

7.11 ? ⓧ



aluminium, of cross section σ and circumference $4a$, density d and resistivity ρ

a) Loop drops until power loss in coil, from induced current, matches loss rate of gravitational potential energy. This occurs at a velocity v:

$$\text{Power} = V^2/R = \frac{(d\Phi/dt)^2}{R} = \frac{(Bav)^2}{R}$$

$$\text{G.P.E. loss rate} = mg\, dy/dt = mgv$$

Solve for v:

$$\frac{B^2a^2v^{\cancel{2}}}{R} = mg\cancel{v}, \quad v = \frac{mgR}{B^2a^2} = g\underbrace{\left(\frac{mR}{B^2a^2}\right)}_{\tau = 2.8(-4)\text{ sec.}}$$

To put in numbers, $\underset{\text{Vol.}}{}$

$$m = d\,(4a)(\sigma)$$
$$R = \rho\,\frac{4a}{\sigma} \leftarrow \text{length} \atop \leftarrow \text{cross section}$$

$$v_{\text{term}} = \frac{d\,(4\cancel{a})\cancel{\sigma}\cdot 4\cancel{a}}{\cancel{\sigma}} \cdot \frac{g\rho}{B^2\cancel{a^2}} = \frac{16dg\rho}{B^2} = g\tau.$$

$d = 2.7\times10^3\,\text{kg/m}^3$, $g = 9.8\,\text{m/s}^2$, $B = 1\,T$, $\rho = 2.65(-8$

$$\underline{v} = \frac{(4)(4)(2.7(3))(9.8)(2.65(-8))}{1} = \underline{11.2\,\text{mm/sec.}}$$

b) OK, have to use $F = ma$:

$$m\,dv/dt = |\textstyle\int i\,d\vec{l}\times\vec{B}| = -\underbrace{\frac{Bav}{R}}_{i}\ \underbrace{a}_{\int dl}\ \underbrace{B}_{B}$$

$$\frac{dv}{dt} = -v/(mR/a^2B^2) = -\frac{v}{\tau}, \quad v = v_{\text{term}}\big(1-\underbrace{e^{-t/\tau}}_{.9}\big)$$

at 90%. $t = \tau/\ln(10) = 3\times10^{-3}$ sec.

P3'

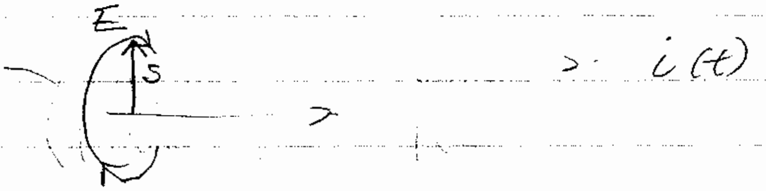
$$B = B_0\cos(\omega t)$$

7.12

Emf in coil $= -d\Phi/dt = \overbrace{B_0 \omega \sin(\omega t)}^{B}\,\overbrace{(\pi a^2/4)}^{A}$

So $\boxed{i = \frac{Emf}{R} = \frac{B_0 \omega\, \pi a^2/4}{R} \sin(\omega t)}$

7.15



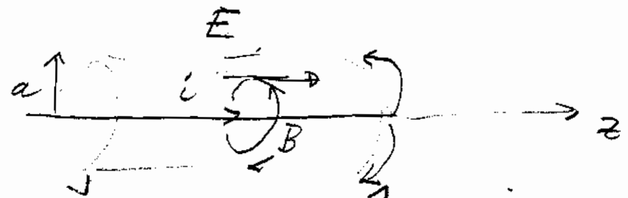
$B = \mu_0 n i$, E-field circulates around flux in $\hat{\phi}$ direction, of size $\oint \vec{E}\cdot d\vec{l} = -d\Phi/dt$,

$$\left.\begin{array}{l} (E)(2\pi s) = \left(\mu_0 n \frac{di}{dt}\right)\pi s^2 \\ \boxed{E = \mu_0 n (di/dt)\, s/2} \end{array}\right\} \quad s < a$$

$$\left.\begin{array}{l} = -(\mu_0 n\, di/dt)\,\pi a^2 \\ \boxed{E = \left(\mu_0 n \frac{di}{dt}\right) \frac{a^2}{s}} \end{array}\right\} \quad s > a$$

P 3'' (Done)

7.16

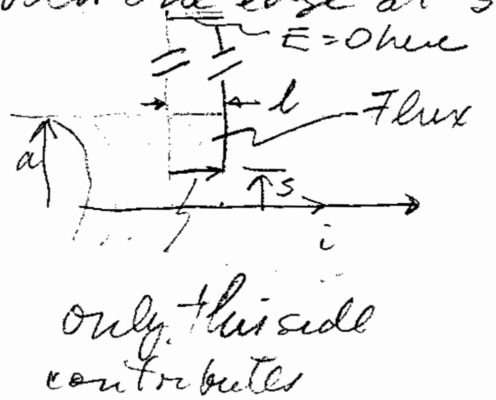


$B = \frac{\mu_0 i}{2\pi s}$, $s < a$

$= 0$, $s > a$.

a) $\vec{B}$, and $d\vec{B}/dt$, are in $\hat{\phi}$ direction, give rise to $\vec{E}$ in $\hat{z}$ direction, longitudinal.

b) Take rectangular loop in plane of page with one edge at $s \to \infty$:



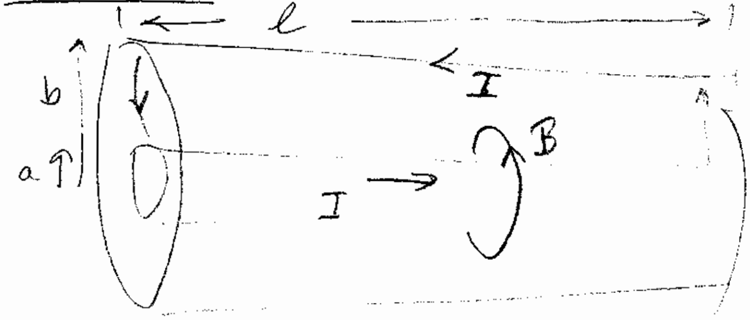
around rectangle $\oint \vec{E}\cdot d\vec{l} = -d\Phi/dt$

$$E l = \frac{d}{dt}\Big(\underbrace{\int_{surface} \vec{B}\cdot d\vec{a}}_{\int_s^a \frac{\mu_0 i}{2\pi s} l\, ds = \frac{\mu_0 l}{2\pi}\ln(a/s)}\Big)$$

or
$$\boxed{\begin{aligned} &\vec{E} = \frac{\mu_0}{2\pi}\left(\frac{di}{dt}\right)\ln(a/s)\hat{z}, \quad s < a \\ &\vec{E} = 0, \quad s > a, \text{ since } \Phi = 0. \end{aligned}}$$

p4 (Start here)

Ex. 7.13: Find "L".



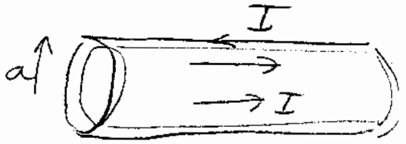
$$\oint B\cdot dl = 2\pi s B = \mu_0 I_{encl.}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$, so

$$\text{Energy} = \int B^2/2\mu_0 \, d\tau = \frac{l}{2\mu_0}\int_a^b \left(\frac{\mu_0 I}{2\pi s}\right)^2 2\pi s\, ds$$

$$= \underbrace{\frac{\mu_0 l}{4\pi}\ln(b/a)}_{L} \quad I^2 \quad = I^2\frac{L}{2}$$

Problem 7.28:



Find L. Plug in b=a, get L=0. Huh? What is wrong? Forgot flux inside wire, which is only contribution here. So need

$$\text{Energy:} \ \frac{l}{2\mu_0}\int_0^a \left(\frac{\mu_0 I}{2\pi}\right)^2\left(\frac{s}{a^2}\right)^2 2\pi s\, ds$$

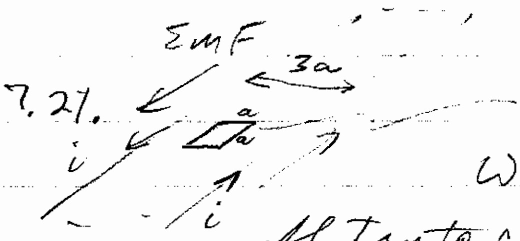
$$= \frac{l\mu_0}{4\pi}\ \frac{a^4/4}{a^4} \quad I^2$$

$$= \frac{\mu_0 l}{4\pi}\ \frac{1}{4}\ I^2 = \ \frac{1}{2}I^2 L$$

$$\boxed{L = \frac{\mu_0 l}{8\pi}}$$

(Go to P6 →)

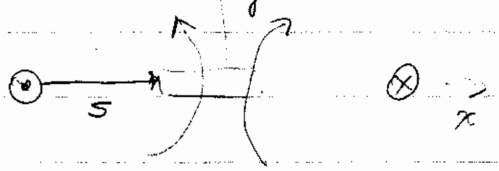
7.27.



$dI/dt = k$

What is EMF in big one?

If Try to calculate $d\Phi/dt$ in big loop. It is a mess! But because $M_{12} = M_{21}$, do not need to: Calculate $d\phi/dt$ in small loop due to I in large loop:



$B_y = \frac{\mu_0 i}{2\pi s}$ from left one

$$\text{So } \Phi_{small} = \int_a^{2a} \frac{\mu_0 i}{2\pi s} a\, ds = \frac{\mu_0 i a}{2\pi} \ln 2$$

From symmetry, right wire gives same, so

$$\Phi_{small} = \frac{\mu_0 i a}{\pi} \ln 2 = \underbrace{\frac{\mu_0 a}{\pi} \ln 2}_{M_{12} = M_{21}} \; i_{large}$$

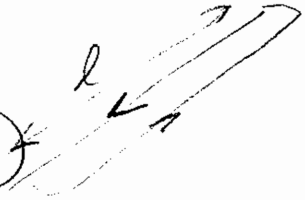
$$\Phi_{large} = \frac{\mu_0 a}{\pi} \ln 2 \; i_{small}, \text{ so}$$

$$\boxed{EMF_{large} = \left(\frac{\mu_0 a}{\pi} \ln 2\right) k}$$

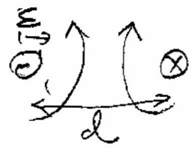
Current induced will fight change in flux. If k is CW in inner loop, CCW EMF in outer one.

p6

7.23



From 7.21,

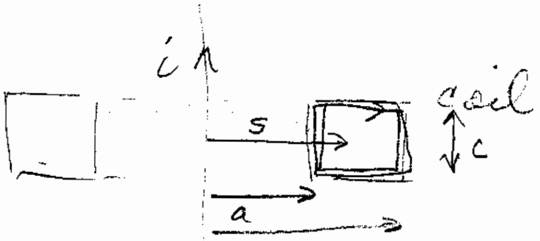


Flux through region between wires is

$$\Phi = \int_{\varepsilon}^{d} \frac{\mu_0 i}{\pi s} l\, ds = \underbrace{\frac{\mu_0 l}{\pi} \ln(d/\varepsilon)}_{L} \quad i$$

So $L \to \infty$ as $\varepsilon \to 0$. The "ε" is important.

(Do not do) 7.24



$i = i_0 \cos(\omega t)$

a) i causes B_ϕ of $\frac{\mu_0 i}{2\pi s}$, linking flux

to Toroid of $\int_a^b \frac{\mu_0 i}{2\pi s} c\, dx = \frac{\mu_0 c i}{2\pi} \ln(b/a)$ per turn

Thus $EMF = -d\Phi/dt = 2 \frac{\mu_0 c}{4\pi} \ln(b/a) \underset{\#\text{turns}}{\underbrace{N}} \underset{di/dt}{\underbrace{\omega i_0 \sin(\omega t)}}$

Using $a = 10^{-2}$, $b = 2 \times 10^{-2}$, $c = 10^{-2}$, $\omega = 2\pi \times 60$, $i_0 = 0.5$ (MKS)

$\mu_0/4\pi = 10^{-7}$, $N = 1000$

$EMF = 2.613(-4) \sin(\omega t)$;

$i_{toroid} = EMF/R = 5.22(-7)\,A \sin(\omega t)$.

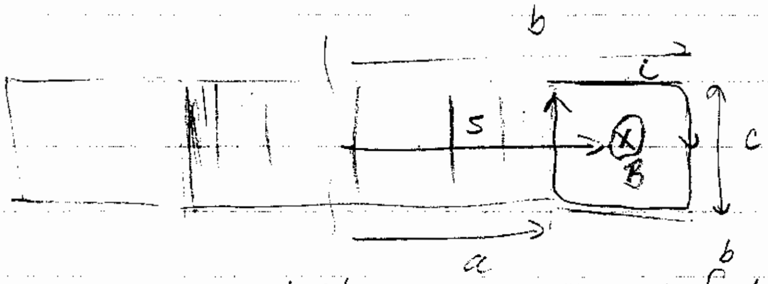
b) Coil has inductance $\frac{\mu_0 c}{2\pi} \ln(b/a) N^2$, so

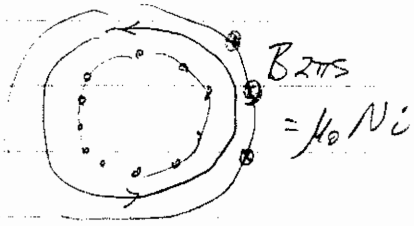
$$\frac{EMF_{dir}}{EMF_{ind}} = \frac{\frac{\mu_0 c}{4\pi} N \ln(b/a) \quad (di/dt)}{\frac{\mu_0 c}{4\pi} N^2 \ln(b/a) \, (di/dt)(1/R)} = 1/NR$$

no. etc.

P7 Do this

Inductance of a Toroid :





$B 2\pi s = \mu_0 N i$

$B = \frac{\mu_0 i N}{2\pi s}$, so $\Phi = N \int_a^b \frac{\mu_0 i}{2\pi s} c\,ds$ $\quad N$

$$= \underbrace{\frac{\mu_0 c N^2}{2\pi} \ln(b/a)}_{L} \; i$$

7.27 Energy stored in toroid : (7.27)

$$\int B^2/2\mu_0 \, d\tau = \frac{1}{2\mu_0} \int_a^b \left(\frac{\mu_0 i N}{2\pi s} \right)^2 (2\pi s)(c)\, ds$$

$$= \frac{\mu_0 i^2 N^2 c}{4\pi} \ln(b/a) = \frac{1}{2} i^2 L$$

so $L = \frac{\mu_0 N^2 c}{2\pi} \ln(b/a)$ ✓

P8

9.1, 9.2, 9.3 ✓, 9.8 ✓, 9.9 ✓, 9.10 ✓, 9.33 ✓

9.1 $f_1 = Ae^{-b(z-vt)^2}$ $f_2 = A\sin[b(z-vt)]$

$f_4 = Ae^{-b(bz^2+vt)}$

a) $\frac{\partial^2 f_1}{\partial z^2} = \frac{\partial}{\partial z}\left(-2b(z-vt)Ae^{-b(z-vt)^2}\right) = \left[-2b + (2b(z-vt))^2\right]Ae^{-b(z-vt)^2}$

$\frac{\partial^2 f_1}{\partial t^2} = \frac{\partial}{\partial t}\left(-2b(z-vt)(-v)Ae^{-b(z-vt)^2}\right) = \left[v^2(-2b+(2b(z-vt))^2\right]Ae^{-b(z-vt)^2}$

So $\ddot{f}_1 = v^2 f_1''$ ✓

b) $f_2'' = \frac{\partial}{\partial z} Ab\cos[b(z-vt)] = -Ab^2\sin[b(z-vt)]$

$\ddot{f} = \frac{\partial}{\partial t}(-v)Ab\cos[b(z-vt)] = -v^2 Ab^2\sin[b(z-vt)]$

So $\ddot{f} = v^2 f''$ ✓

c) $\partial^2 f_4/\partial z^2 = \frac{\partial}{\partial z}(-2b^2z)Ae^{-b(bz^2+vt)} = \left[-2b^2 + (2b^2z)^2\right]Ae^{-b(bz^2+vt)}$

$\frac{\partial^2 f_4}{\partial t^2} = \frac{\partial}{\partial t}(+bv)Ae^{-b(bz^2+vt)} = (bv)^2 \quad Ae^{-b(bz^2+vt)}$

$\frac{\partial^2 f}{\partial t^2} - v^2\frac{\partial^2 f}{\partial z^2} = \left[-2b^2 - (2b^2z)^2\right]v^2 + b^2v^2 \quad Ae.$

9.2 $f = A\sin(\overset{a}{kz})\cos(\overset{b}{kvt})$

Use $2\sin a\cos b = \sin(a+b) + \sin(a-b)$

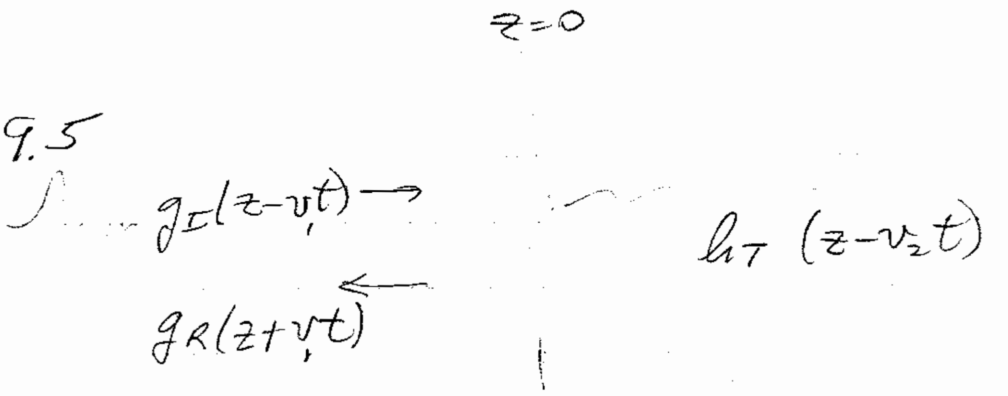
So $f = \frac{A}{2}\left[\sin\left(\frac{kz+kvt}{2}\right) + \sin\left(\frac{kz-kvt}{2}\right)\right]$

$$f = \frac{A}{2}\left[\underbrace{\sin\left(\frac{k}{2}(z+vt)\right)}_{\text{to left.}} + \underbrace{\sin\left(\frac{k}{2}(z-vt)\right)}_{\text{To right}}\right]$$

Since is $f(z+vt) + f(z-vt)$, satisfies wave eq.

P9

9.5



Boundary conditions: at $z=0$

(1) $g_I(-v_1t)+g_R(v_1t) = h_T(-v_2t)$

(2) $\frac{\partial}{\partial z} g_I(-v_1t) + \frac{\partial g_R}{\partial z}(v_1t) = h_T(-v_2t)$

Note $\frac{\partial g_I}{\partial t} = -v_1\frac{\partial g_I}{\partial z}$, $\frac{\partial g_R}{\partial t} = v_1\frac{\partial g_I}{\partial z}$, $\frac{\partial h_T}{\partial t} = -v_2\frac{\partial h_T}{\partial t}$

So second equation becomes

$$-\frac{1}{v_1}\left[\left(\frac{\partial g_I}{\partial t}\right)-\left(\frac{\partial g_R}{\partial t}\right)\right] = -\frac{1}{v_2}\left(\frac{\partial h_T}{\partial t}\right)$$

or, $\int dt \rightarrow$

(2) $\frac{1}{v_1}(g_I - g_R) = \frac{1}{v_2}h_T + C$ ← some constant of integration.

Solving (1) + (2),

$$\left.\begin{aligned} g_I + g_R &= h_T \\ g_I - g_R &= \frac{v_1}{v_2}h_T + C v_1 \end{aligned}\right\}$$

$$2g_I = h_T(1+v_1/v_2) + Cv_1$$

$$h_T = \frac{2g_I - Cv_1}{(1+v_1/v_2)} = \frac{2v_2}{v_2+v_1}g_I - C\frac{v_1v_2}{v_2+v_1}$$

$$g_R = \left(\frac{2v_2}{v_2+v_1} - 1\right)g_I - C\frac{v_1v_2}{v_2+v_1}$$

$$\boxed{\begin{aligned} g_R(v_1t) &= \left(\frac{v_2-v_1}{v_2+v_1}\right)g_I(-v_1t) - \left.C\frac{v_1v_2}{v_2+v_1}\right\} K \\ h_T(-v_2t) &= \left(\frac{2v_2}{v_2+v_1}\right)g_I(-v_1t) - \frac{Cv_1v_2}{v_2+v_1} \end{aligned}}$$

$-v_1t$

$+\frac{\tau v_1}{v_2}$

$\tau = -v_2t$

Set the terms "K" = 0, so then

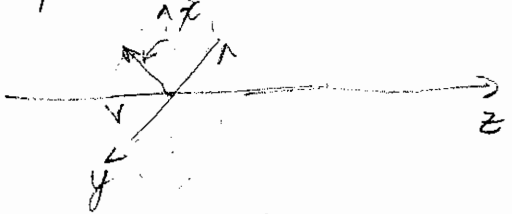
Note: The arguments are not just "t" but related to t, so, for example, at time τ,

$$g_R(\tau) = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) g_I(-\tau)$$

$$h_T(\tau) = \left(\frac{2 v_2}{v_2 + v_1}\right) g_I\left(\frac{v_1}{v_2}\tau\right)$$

(PS, sorry, I called the reflected beam g and the transmitted beam h, unlike the text.)

9.8 $\tilde{f} = \tilde{A}\left(e^{i(kz-\omega t)}\,\hat{x} + e^{i\frac{\pi}{2}}e^{i(kz-\omega t)}\,\hat{y}\right)$

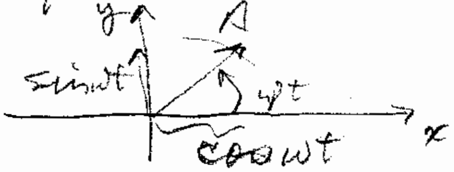


a) at $z=0$, $\vec{f} = A\,\mathrm{Re}\left(\hat{x}\,e^{-i\omega t} + i\,\hat{y}\,e^{-i\omega t}\right)$

$$\vec{f} = A\left(\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t)\right)$$

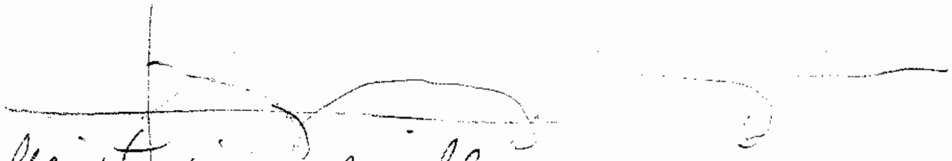
Using ($\sin(-\omega t) = -\sin(\omega t)$, $\cos(-\omega t) = \cos(\omega t)$)

a vector of length A circulating about z axis



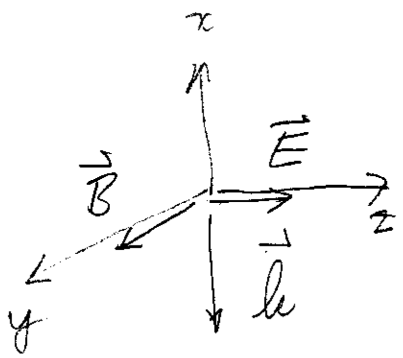
obeys the same eq'ns. If you look toward direction of wave, goes CCW.

b) Wave vector is a helix.



c) Shake it in a circle.

9.9 a)



Can do by inspection.
Since $\hat{k} = \hat{E}\times\hat{B}$
$\vec{B}$ along $\hat{y}$.

$$\vec{E} = \hat{z}\, E_0\, e^{i(-kx-\omega t)}$$
$$B = \hat{y}\,\frac{E_0}{c}\, e^{i(-kx-\omega t)}$$

b) Cannot do by inspection.

$$\boxed{\vec{k} = (\hat{x}+\hat{y}+\hat{z})\frac{k}{\sqrt{3}}}$$ (So $|\vec{k}| = k$)

$\hat{n}$ is in xz plane so $\hat{n} = a\hat{x} + b\hat{z}$

Choose a & b so $\hat{n}\cdot\hat{k} = 0$ or $\boxed{\hat{n} = (\hat{x}-\hat{z})/\sqrt{2}}$

$a + b = 0$ or $a = -b$, or

($|\hat{n}| = 1$).

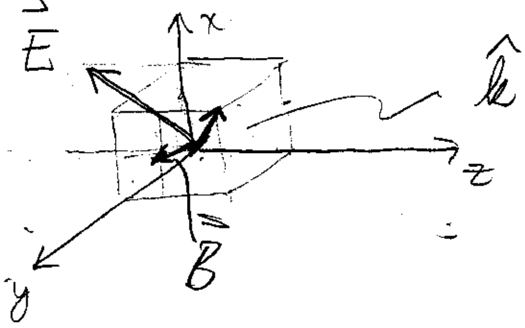
$$\boxed{\vec{E} = \frac{(\hat{x}-\hat{z})}{\sqrt{2}} E_0}$$

Get B from $\hat{k}\times\vec{E} = \vec{B}\omega \Rightarrow$

$$\boxed{\vec{B} = \frac{k}{\omega}\left(\frac{1}{\sqrt{6}}\right)(-\hat{x}+2\hat{y}-\hat{z})\,E_0}$$

~~So~~ $$\vec{E} = \left(\frac{\hat{x}-\hat{z}}{\sqrt{2}}\right)E_0\, e^{i\left[\frac{k}{\sqrt{3}}(x+y+z)-\omega t\right]}$$

$$\vec{B} = \frac{E_0}{c}\left(\frac{-1}{\sqrt{6}}(\hat{x}-2\hat{y}+\hat{z})\right)e^{i\left[\frac{k}{\sqrt{3}}(x+y+z)-\omega t\right]}$$



Can you visualize this?
Good luck - - .

P13

9.10. a) The force/Area = Momentum/area-sec

$$= \frac{S}{c} = \frac{1.3\times10^{3}\ W/m^{2}}{3\times10^{8}\ m/s} = \underline{4.3\times10^{-6}\ N/m^{2}}$$

b) If a perfect reflector, ×2 → $\underline{8.6\times10^{-6}\ N/m^{2}}$

c) 1 atm = $\underline{10^{5}\ N/m^{2}}$ so

Radiation pressure ≃ $4\underline{\times10^{-11}}$ atm.

(or 8×10^{-11})

P14

9.33 Let $\boxed{\vec{E} = A\overbrace{\frac{\sin\theta}{r}\, e^{i(kr-\omega t)}}^{E_\theta}\,\hat{\theta}}$ (Drop the A, put in at end)

① $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t = +i\omega\vec{B}$

$$= \frac{1}{r}\left(\frac{\partial}{\partial r} rE_\theta\right)\hat{\phi} = \frac{1}{r}\frac{\partial}{\partial r}\left(\sin\theta\, e^{i(kr-\omega t)}\right)\hat{\phi}$$

$$= ik\frac{\sin\theta}{r}e^{i(kr-\omega t)}\hat{\phi} = +i\omega\vec{B}$$

$$\boxed{\vec{B} = \frac{k}{\omega}A\frac{\sin\theta}{r}e^{i(kr-\omega t)}\,\hat{\phi}}$$

②a) Does $\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\,\partial\vec{E}/\partial t$? $= \mu_0\epsilon_0(-i\omega)\vec{E}$

$$\vec{\nabla}\times\vec{B} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{k}{\omega}\frac{\sin^2\theta}{r}e^{i(kr-\omega t)}\right)\hat{r}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k}{\omega}\sin\theta\, e^{i(kr-\omega t)}\right)\hat{\theta}$$

$$= \boxed{\frac{2}{r^2}A\frac{k}{\omega}\cos\theta\, e^{i(kr-\omega t)}\,\hat{r}} \to 0 \text{ as } r\to\infty$$ Ummm - - -

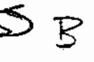
$-\frac{1}{r}\frac{k}{\omega}\sin\theta\,(ik)\,e^{i(kr-\omega t)}\,\hat{\theta}$, so if we drop the radial part (but it is really there!)

$$\mu_0\epsilon_0(-i\omega)A\frac{\sin\theta}{r}e^{i(kr-\omega t)}\hat{\theta} \overset{?}{=} -\frac{k}{\omega}\frac{A\sin\theta}{r}(ik)e^{i(kr-\omega t)}$$

$$\mu_0\epsilon_0\cancel{\omega} \overset{?}{=} \frac{k}{\omega}\frac{k}{\omega} = \frac{1}{c^2}$$, ✓. Works.

But only if we drop the $1/r^2$ term.

b) $\vec{\nabla}\cdot\vec{B} = 0$ because only B_ϕ exists and is independent of ϕ (Lines of B are continuous



c) $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$, ✓ we used this to get $\vec{B}$.

d) $\vec{\nabla}\cdot\vec{E} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r}e^{i(kr-\omega t)}\right)$

$$= \frac{2}{r^2}A\cos\theta\, e^{i(kr-\omega t)} \neq 0!$$

③ What is $\vec{S}$? $\hat{\theta} \times \hat{\phi} = \hat{r}$, so points radially out. $\vec{S} = \frac{1}{\mu_0}(\mathrm{Re}\,\vec{E} \times \mathrm{Re}\,\vec{B})$

$$= \frac{1}{\mu_0} A^2 \frac{k}{\omega} \frac{\sin^2\theta}{r^2} \cos^2(kr-\omega t)$$

$$\boxed{\vec{S} = c\,\varepsilon_0 A^2 \frac{\sin^2\theta}{r^2} \cos^2(kr-\omega t)\,\hat{r}}$$

④ Total power, cycle averaged, is

$$P = \int I(\theta)\, r^2 d\Omega = \frac{c\varepsilon_0 A^2}{2} \int_{-1}^{1}\int_{0}^{2\pi} \frac{\sin^2\theta}{r^2} r^2\, d(\cos\theta)\, d\phi$$

$$= \frac{c\varepsilon_0 A^2}{2}\, 2\pi \underbrace{\int_0^\pi \sin^3\theta\, d\theta}_{-\frac{1}{3}\cos\theta(\sin^2\theta+2)\Big|_0^\pi}$$

$$-\frac{2}{3}(-1-1) = 4/3$$

$$\boxed{P = \frac{4}{3}\pi c\varepsilon_0 A^2}$$

Note, A has dimensions of (E-field)×(distance).