Hw- 4

4.1. (a) I have an ensemble of pure states of identical spin 1/2 particles, and I measure the spin of each of these particles along the z-axis and found that the average value is zero. In such a single measurement I cannot determine the nature of the state vector. Describe the class of states that will have $<S_z>=0$.

(b) Suppose I also measure spin along the z-axis (not sequentially but independently) and also found that $<S_x>=0$.

Can these two independent measurements tell me the spin state vector uniquely now? If not, what do I need to do to uniquely find the state vector ?

Note that we need to do many independent measurements.

4.2. (a) What does it mean that two particles are entangled?

(b) If two spin ½ particles are given by

$$|\Phi_A\rangle = \frac{1}{2}\left(|++> - |+-> + |-+> - |-->\right)$$

Are they entangled?

(c) How about if they are given by

$$|\Phi_B\rangle = \frac{1}{2}\left(|++> + |+-> - |-+> + |-->\right)$$

4.3. If two spin ½ particles are created at t=0 and they are given by an entangled state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|++> + |-->\right)$$

where the spin states are referred to the z-axis quantization. These two particles then fly apart. When they are 100 meters away from each other we will make the measurements. I will meaure particle 1 and

you will measure particle 2. You will always measure the spin along the z-axis. I may use z-axis or I may use x-axis. I will make the measurement first.

(a) If I measure along the z-axis and I got $+\hbar/2$, what will you get after me? What is the state vector after your measurement.

(b) Instead (a), if I measure along the x-axis and I got $+\hbar/2$, show that you will get either $+\hbar/2$ or -$\hbar/2$, with equal probability.

(c) Suppose you get $+\hbar/2$ in (b), what is the state of the two particles after your measurement?

(d) If next I measure the spin along the z-axis **again**, what will I get?

(e) If later you measure the spin, what will you get?