

### Problem set-9

**29. Hydrogen atom in a magnetic field.** In this exercise we will assume that the magnetic field is rather strong such that the Zeeman effect is larger than the spin-orbit interaction. This is **the Paschen-Back limit**.

(a) First consider the perturbation from the magnetic field. Starting with 2p state in hydrogen, identify all the degenerate states with the proper quantum number using  $|m_\ell m_s\rangle$ . Calculate the energy shift of each state due to magnetic field and sketch the relative energy order. Indicate states that are degenerate.

(b) We next include effect from the spin-orbit interaction. Use first order perturbation theory to calculate the shift in energy due to the spin-orbit interaction. Is the degeneracy removed after the spin-orbit interaction is included? (To simplify your work, you do not have to evaluate the radial integral, just use some symbols to represent the radial integrals.)

(c) When the spin-orbit and the Zeeman effect are of comparable size, you need to treat both terms on equal footing. You need to start with a "basis set". You can choose a basis set in which spin-orbit interaction is diagonal. The basis set then would be  $|\ell s j m\rangle$  or  $|j m\rangle$  if we focus on 2p only, and  $j=1/2$  and  $3/2$  only. In a magnetic field,  $j$  is no longer a good quantum number. Use commutation relation to show that  $J_z$  still commute with the whole Hamiltonian and thus  $m$  is still a good quantum number. Identify  $|j m\rangle$  states that can be mixed by the magnetic field. In this case, construct the matrix that you need to diagonalize to find the new eigenstates. (Again, work out the angular part only.)

### 30. Van der Waal interaction between two hydrogen atoms.

Two hydrogen atoms are separated by a distance  $R$  which is large compared to the radius of the hydrogenic ground state. Assume that spin can be neglected, write down the Hamiltonian for the two electrons. Measure the position of each electron from its center at the nucleus. Let  $H_0$  be the Hamiltonian of two noninteracting hydrogens. Clearly the zero order wavefunction for the two atoms is  $u_{100}(r_1)u_{100}(r_2)$  where  $u_{100}$  is the ground state 1s wavefunction with  $r$  given from its respective nucleus A or B.

(a) Identify  $H'$ -- the perturbation.

(b) Assume that  $R$  is large, show that to first order  $H'$  is given by

$$H' = \frac{e^2}{R^3} (x_1 x_2 + y_1 y_2 - 2z_1 z_2)$$

where the coordinates of the electron are measured from its respective center and  $z$  is the axis along the internuclear line.

(c) Show that the first order correction to the energy is zero. Explain why the Van der Waals potential at large distance is given by  $C/R^6$  and that  $C$  is negative.

### 31. Dipole moment of an atom.

(a) Show that if a state has well-defined parity then the dipole moment is zero.[Note: In the electroweak theory parity is no longer a good quantum number. Thus an atom in general would have a very small dipole moment.]

(b) In the  $n=2$  excited states for hydrogen, the  $2s$  and  $2p$  states are degenerate. The degeneracy is removed by the external electric field. Use first order perturbation theory to find the new eigenenergies--neglect electron spin. Sketch how the energies are split vs the electric field.

(c) Show that for  $m=0$  the two new states are given by  $(u_{200} \pm u_{210})/\sqrt{2}$  no matter what the field is. Sketch these two wavefunctions and explain why they have different eigenenergies.