27. **Another application of angular momentum addition and Wigner-Eckhart Theorem**:

(a) The 2p state of a hydrogen atom can have j=3/2 and j=1/2 if the spin-orbit interaction is included. Each $|\,jm>$ state is the result of coupling $\ell = 1$ and spin $s = \frac{1}{2}$. Use Y_{1m} to represent the orbital wavefunction and α and β to represent the spin up and spin down, respectively. Write down explicitly the angular wavefunction for all the $|\,jm>$ states in terms of linear combinations of products of orbital and spin functions. (Use Clebsch-Gordon coefficients from your favored sources.)

(b) Write down the ground state wavefunction including the spin. Note that the ground states can be expressed as $|\,j'm'>$ with $j' = \frac{1}{2}$ and $m' = \pm\frac{1}{2}$.

(c) The ground state can be excited to the 2p state with j=3/2 and j=1/2 by absorption of a photon. The transition operator is a dipole operator which is proportional to Y_{1q} with q=0, ± 1. The transition rates from an initial state $|\,j'm'>$ to a final state $|\,jm>$ is proportional to $|< jm\,|\,Y_{1q}\,|\,j'm'>|^2$. Calculate the transion rate from each initial to each final state. What is the total rate for transition to j=3/2 as compared to transition to j=1/2 final states. Assume that initially the m'=$\pm\frac{1}{2}$ are equally populated.

(Note: you can calculate the relative rates at this point only. Later we will show how to calculate the absolute rates.)

28. Problem 17.6 of Merzbacher. (p.449)-- This one should be rather easy by now.