Problem set-6

21. **Practice the use of rotational matrices**.

An eigenfunction $Y_{20}(\theta,\phi)$ which is the eigenstate of L^2 and L_z. What is the eigenfunction of the same state if it is an eigenstate with respect to the new z' axis which is rotated by an angle $\beta=30^0$ about the y axis? Use the rotation matrix to obtain the new eigenfunction and show that it is indeed the eigenfunction of the $L_{z'}$. Note that if the angle $\beta=90^0$ you would obtain the eigenfunctions of L_x and L^2.

You need the rotational matrix. You can find them in some books, for example, in Edmond's p.57, (Augular Momentum in Quantum Mechanics) or from some programs.

22. **Lifetime of positronium in a magnetic field**.

Problem 17.4 in Merzbacher.

23. **Rotational eigenstates of a symmetric top molecule**.

You will start with problem 17.7 of Merzbacher. I do not want you to do part (a) and (b). Assume that part (a) is already given. Do part (c) and (d).

Then do problem 17.8 to get the eigenenergies. Assume that I_1 is greater than I_3, sketch the energy levels for each given m for the lowest five values of J's. Consider m=0,1,2,3. This will give you the typical rotational levels structure of a symmetric top like that of a water molecule or an ammonia molecule to the first order.

24. **Integrals involving three spherical harmonics**. Evaluate the integral

$$\int Y_{\ell_1 m_1}(\theta,\phi) Y_{\ell_2 m_2}(\theta,\phi) Y_{\ell_3 m_3}(\theta,\phi) d\Omega$$

and show that it is proportional to $(-1)^{m_3} < \ell_1 \ell_2 m_1 m_2 | \ell_3 - m_3 >$, the proportional constant is independent of the magnetic quantum numbers. Determine this coefficient. Note: Use Wigner-Eckhart theorem and properties of angular momentum coupling.