

Problem set-11

36. Rutherford scattering involving identical charged particles.

Calculate the differential cross sections in the center-of-mass frame for the collision between ^{12}C on ^{12}C , ^{12}C on ^{13}C and ^{13}C on ^{13}C at the same center of mass energy of 5 MeV. Plot the angular distributions on the same graph for the three collisions.

Note: At the low energy considered only elastic scattering is needed. You need to use the analytical scattering amplitudes for Coulomb scattering derived earlier or from any textbooks. Experimental data can be found in Phys. Rev. Lett. 4, 365 (1960), for example.

37. Constructing two-electron wavefunctions using determinants.

Consider the $1s2s\ ^1\text{S}$ and $1s2s\ ^3\text{S}$ states of He. We want to write down the total wavefunctions which are eigenstates of L^2 , S^2 , L_z and S_z and that they satisfy the correct exchange symmetry. There are two ways to do it.

(1) The first method is to construct functions which are eigenstates of L^2 , S^2 , L_z and S_z and after that introduce the symmetrization. Let $u = \phi_{100}$ and $v = \phi_{200}$, where the radial and angular functions are included. For the spatial part, the two-electron function is $u(1)v(2)$ before symmetrization. They can be made symmetric or antisymmetric as $u(1)v(2) \pm u(2)v(1)$. For the spin part, we know the symmetric or antisymmetric ones are $\alpha(1)\beta(2) \pm \alpha(2)\beta(1)$ where I did not normalize it. From these construct the wavefunctions for $1s2s\ ^1\text{S}$ and $1s2s\ ^3\text{S}$ states of He.

(2) You can also construct the proper wavefunctions using the determinants of spin-orbitals as in the standard textbooks. To do that you can use $u\alpha$ as the spin-orbital for electron 1 and $v\beta$ as the spin-orbital for electron 2. You get proper antisymmetric wavefunction by constructing a determinant. The resulting function has the correct symmetry but it is not an eigenfunction of S^2 . Clearly in this example, you can start with the spin-orbitals $u\beta$ and $v\alpha$ as well to construct another determinant. These two determinants would have identical energy. Thus you may expect that the plus and minus linear combinations of the two determinants can be shown to be the eigenstates as well and they are eigenfunctions of S^2 . Show that the results from this method agree with what you obtained from part (1).

38. The energy levels of $n=2$ excited states of atomic hydrogen.

(a) If the electron only sees the Coulomb potential from the nucleus, the 2s and 2p levels are degenerate. Identify all the states in the $n=2$ manifold and all the quantum numbers for each state. You need to include the spin of the electron. Clearly you have 8 states in this manifold.

(b) After you include the spin-orbit interaction, some of the states are separated and the degeneracy is removed. Identify all the eight states again and their good quantum numbers. Group the degenerate states together.

(c) In a magnetic field, from (b) how the levels are now split? Assume that the B field is weak. Label all the eight states and identify states that are degenerate.

(d) If a weak electric field is applied (no magnetic field), how the levels are split? What are the good quantum numbers for each state and identify states that are degenerate.

You are to use the concept of perturbation theory, but no calculations are required.

39. Application of sudden approximation.

A hydrogen atom initially is in the $n=1$ ground state. Suddenly by β -decay the nuclear charge changes to $Z=2$. What is the probability that the electron will change to $n=1$ of $\text{He}^+(1s)$.