Problem set I:

In working out problems you do not need to derive every equations you use. You can start with, say, eq. (12.11) of Merzbacher or from any other books. Write down the solutions as you understand them, but be ready to explain.

1. Assume that the potential energy of the deuteron is given by $V(r)=-V_0,\ r<r_0;$ $V(r)=0\ r^3 r_0$.

   (a) Show that the ground state of the deuteron possesses zero orbital angular momentum (l=0). In other words, in any other angular momentum the binding energy will be less.

   (b) Assume that l=0 and estimate the value of V_0 under the additional condition that the value of the binding energy is much smaller than V_0.

   (c) The binding energy of the deuteron is 2.2 MeV, estimate the depth of the potential, expressed in MeV.

2. A particle of mass m is bound in a 3-*dimensional* isotropic harmonic oscillator potential with a spring constant k.

   (a) Write the Schroedinger equation for this system in Cartesian and spherical coordinates.

   (b) In Cartesian coordinates calculate the energies of the first three lowest levels and identify the degeneracy of each level. For the second lowest level, identify if each state is an eigenstate of the parity operator.

   (c) Write down the radial equation in spherical coordinates. For the second level, based on the solutions obtained from Cartesian coordinates, obtain new eigenstates that are eigensolutions of L^2 and L_z as well. What is the parity of each state?

3. We can treat the H_2^+ molecule as consisting of an electron moving in the field of two stationary protons. Let the origin of the coordinate frame be set at the midpoint of the internuclear axis,

   (a) write down the Hamiltonian for the electron-- the two protons are separated by R.

   (b) Show that $[H, L_z]=0$ but $[H, L^2]\neq 0$. Also that parity is a good quantum number. Choose any plane that contains the internuclear axis, show that the reflection with respect to this plane is also a symmetry operation.

4. The rotational motion of a homonuclear diatomic molecule such as H_2 and I_2 can be treated like a rigid rotor.

(a) Write down the Hamiltonian for the rotational motion of such a rigid rotor. If the internuclear separation is R, and the mass of each atom is m, what is the moment of inertia I?

(b) Write down the general expression of the energy levels.

(c) The internuclear separation of H_2 is 0.74 ^{o}A and for I_2 is 2.67 ^{o}A, calculate the rotational energy of the first state, in electron volts. Estimate the rotational period of each molecule and express them in picoseconds (10^{-12}s).